

# AMSAA



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## DEVELOPMENT OF A *MATHEMATICA* TOOL FOR IMPLEMENTATION OF PROGNOSTICS BASED ON LIFE HISTORY

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13. ABSTRACT (Maximum 200 words) In order to increase mission reliability and reduce the logistics footprint, considerable interest is now being focused on the implementation of prognostics. One approach to prognostics is to track usage in terms of miles, hours or cycles, and generate replacement-before-failure rules for components subject to aging whenever the system is preparing to enter a period during which failures must be zealously avoided (e.g., deployments or combat pulses). This report documents the development and notional application of a new tool that implements this approach. The tool, which is an extension of <i>Mathematica</i> , generates graphs and tables for a variety of metrics that one could use in an interactive decision-making process. <i>Mathematica</i> is a leading commercial software package for performing mathematics. Key chapters in this report constitute a basic set of electronic templates for applying the new tool. The tool itself is provided in the appendices.				
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John A. Sereno (Army Evaluation Center)

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# Chapter 1

## Introduction

### A Broad View of Prognostics

In order to increase the mission reliability and reduce the logistics footprint of new Army systems and equipment, considerable interest is now being focused on the implementation of prognostics. In this report, the term prognostics refers to embedded algorithms and sensors that cue the operator and/or maintainer to an approaching need for maintenance so that it can be performed before failures occur. Figure 1-1 depicts this broad view of prognostics.

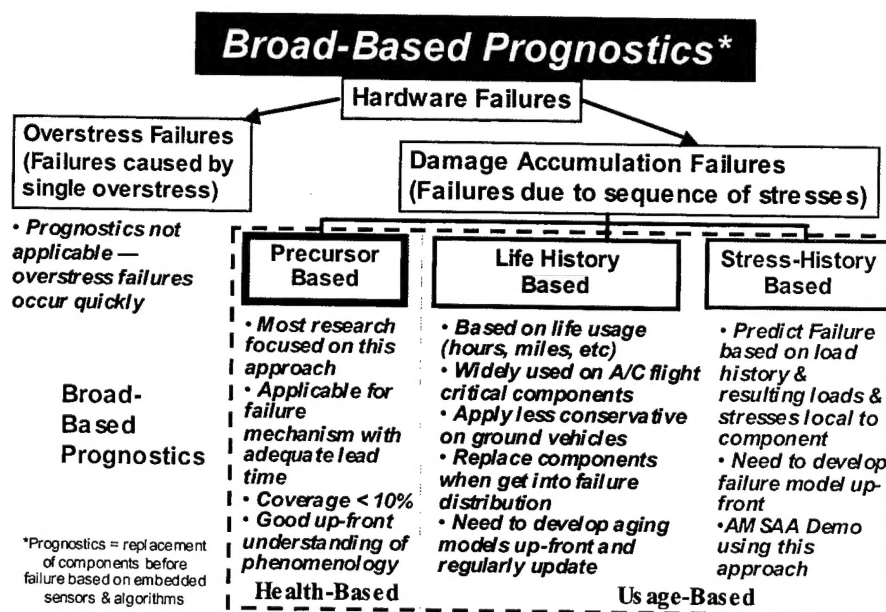


Figure 1-1

Hardware failures can be classified as either overstress failures (e.g., the failure of a television due to an electrical transient resulting from a nearby lightning strike) or damage-accumulation failures (e.g., fatigue cracking of a solder connection due to temperature-cycling stresses). Prognostics is not applicable to overstress failures because such failures typically occur immediately after the overstress event. Many failures of Army equipment are due to the gradual accumulation of damage due to cyclic stresses applied over time. We have identified three approaches to performing maintenance before damage-accumulation failure mechanisms occur:



1. The first we term “precursor-based” failures. With this approach, one monitors performance degradation, and maintenance is triggered when degradation drops to a specific level. It is difficult to determine which parameter(s) to monitor for a particular component and what the maintenance threshold should be. University researchers applying this approach to a single failure mode can typically predict failures just minutes ahead of time. The consensus of the industry-Army Future Combat System Reliability Availability Maintainability Working Group is that, in the near- and mid-term, this approach to prognostics will cover less than 10% of failure modes with lead time sufficient to avoid failures during combat pulses.
2. The life-history approach involves the tracking of usage in terms of component hours, miles or cycles, and the use of a probabilistic lifetime model such as the Weibull or lognormal distribution. Maintenance can be performed when the risk of failure becomes too great. This approach, while not as accurate as the previous one, is appropriate for components where a PoF analysis is not available or where multiple suppliers are involved.
3. The PoF-based approach involves detailed tracking of component load/stress history, in conjunction with the use of a physics-of-failure (PoF) model, in order to predict how much life remains. Maintenance can be performed when the risk of failure becomes too great. This approach requires that a PoF analysis be performed on the component first. It is ideal for structural and other components that are not expected to be replaced by components whose construction varies due, for example, to multiple suppliers being involved.

Figure 1-2 depicts the "Beyond Legacy Reliability Availability Maintainability (RAM) Practices Scorecard".

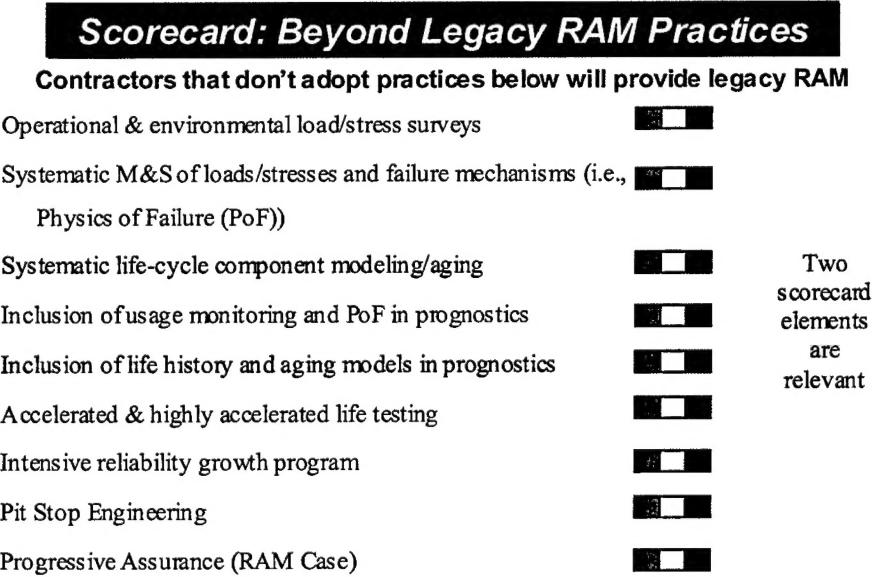


Figure 1-2

The practices on this “scorecard” are based on those identified in recent years by the Army RAM Panel tasked by GEN Kern. The scorecard is a handy tool for identifying the extent to which "beyond legacy" practices are being used in our acquisitions. If the practices on the scorecard are not implemented by our contractors, we can expect to achieve legacy

reliability, which will require the legacy logistics footprint and its associated deployability requirements. The two highlighted elements are relevant to prognostics:

- “Systematic life-cycle component modeling/aging” requires that the contractor analyze and model components that may cause system aborts and essential function failures. This provides the analytical underpinning as well as component models and parameters for implementation of prognostics based on life history.
- “Inclusion of life history and aging in prognostics” is a subsequent, prognostics-implementation step where the contractor must address the possibility of replacing components that are subject to aging before they fail.

At this point in time, little exists in terms of tools or documentation on how to implement prognostics based on life history. This technical report documents a first step towards implementation of life-history based prognostics.

## Approach

The first element of our approach is motivated by replacement rules for flight-critical components. Such replacement rules are widely applied to flight-critical components that are subject to damage accumulation or aging. Failures of these components cannot be tolerated because of safety-of-flight considerations. Flight-critical components that age are generally modeled with either a Weibull or a lognormal distribution and highly-conservative replacement rules are then developed. For example, a replacement rule may be established at the quantity of flight hours where the probability of a new component surviving equals 0.9999. This is depicted in Figure 1-3.

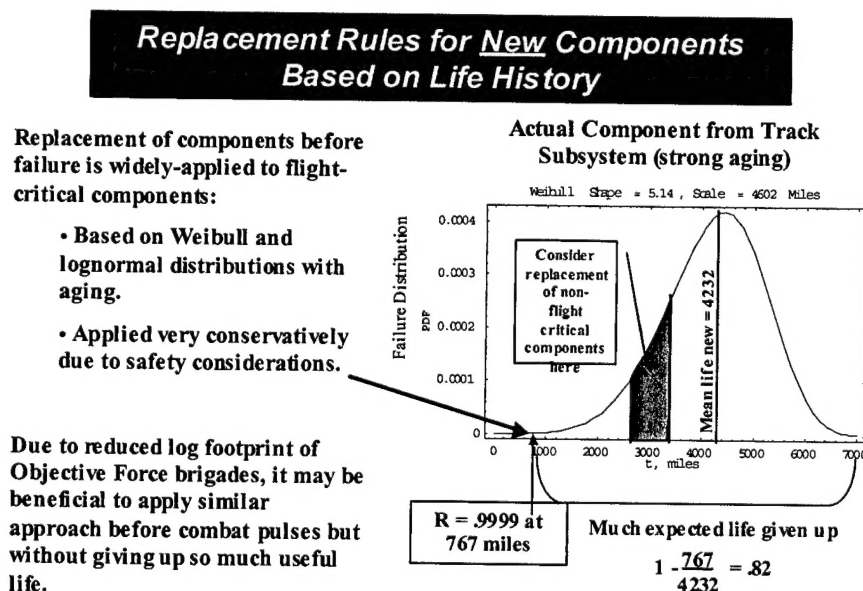
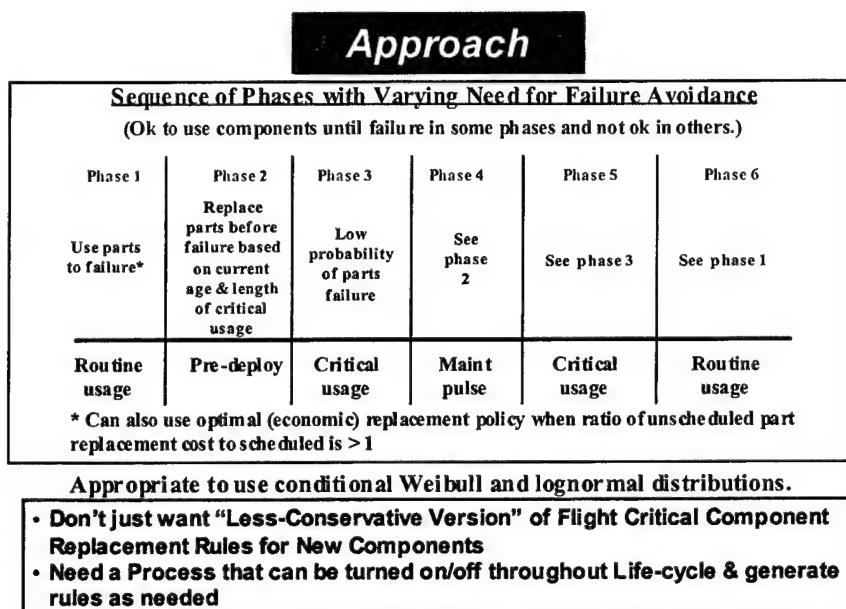


Figure 1-3

The component used in Figure 1-3 is an actual track component that experienced fairly strong aging. The component's life distribution is modeled with a Weibull distribution with the parameter values indicated. As the graph illustrates, use of a conservative, flight-critical rule would cause the replacement of the components very early in its life cycle when the risk of failure is still extremely low. In this example, 82% of the useful life of the component is given up. One need not be so conservative when working with components that are not flight critical. One can consider replacing such components a bit later in the cycle as indicated by the green region in the graph above. This would place the component age closer to the mean, while still avoiding high failure risks.

The second element of our approach stems from a recognition that it is more important to avoid failures during some portions of the life cycle than others. We anticipate that Army systems will undergo a sequence of phases as illustrated in Figure 1-4.



**Figure 1-4**

While it may be reasonable to use components until failure in some life-cycle phases, it is not okay to do so in others. In the first phase above, the system is subject to routine usage and components that age need not be replaced until they fail. Perhaps the system will soon enter a critical-usage phase, such as a deployment, and failures will need to be zealously avoided. In the pre-deployment phase, one would like to be able to rank aging components based on their current ages and the amount of usage expected to be encountered during the critical-usage phase and replace the components most likely to fail. Following the critical-usage period, aging components could be ranked once again before a subsequent critical-usage period. In phase 6, the systems returns to routine usage.

Due to the varying need for failure avoidance, we did not just want a less-conservative version of flight-critical component replacement rules for new components. To handle these sequences, a process is needed that could generate replacement rules as needed by operators or maintainers, and that can be turned on and off throughout the life cycle. We set out to build a tool that will enable the implementation of just such an approach.

## Conditional Distributions

When Weibull and lognormal distributions are used to model component reliability, one generally assumes that the component is new. This is assumed when formulating replacement rules for flight critical components. But this is not generally the case for components in a vehicle about to undergo a critical-usage phase. A more general approach is to use a conditional distribution which adjusts the original distribution based on the age of the component at the start of the critical-usage period. This actuarial approach is used to calculate life insurance premiums. For example, life insurance premiums for a 10-year policy are more expensive for a 60 year old than for a 20 year old.

A conditional distribution for the track component is depicted in Figure 1-5. The curve that starts at zero miles corresponds to a new component. At an early age, a new component will have a low failure probability. This is the low-risk tail of the distribution. Thereafter, the failure probability gradually starts to increase as the component ages. The right-most curve in the graph corresponds to a component that already has 6,000 miles on it but has not failed. This curve does not have a low-risk tail and failure probability accumulates rapidly due to the steep slope of the curve. If the system was preparing for a critical-usage period, it would probably be best to replace such components if they had survived to an age of 4,000 - 6,000 miles.

### Conditional Distributions

When Weibull and lognormal distributions are used, one generally assumes the item is new.

One exception is life insurance – not everyone is assumed to be a newborn.

- Premiums for a 10-year policy is much more expensive for a 60-year old than a 20-year old.

Need embedded models, on-board sensors, replacement metrics and interactive decision-making process to implement this approach on non-flight critical Army components.

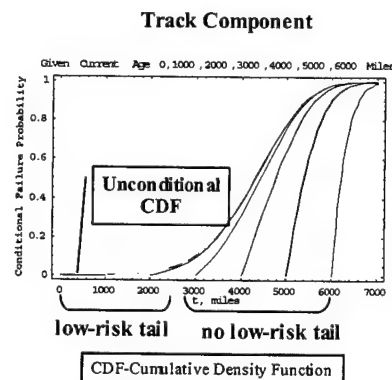


Figure 1-5

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## Structure of This Report

This technical report documents the development of a *Mathematica* tool that implements conditional forms of the Weibull and lognormal distributions. Software which includes the conditional forms of these distributions is not readily available. *Mathematica* ships with a standard add-on package `Statistics`ContinuousDistributions`` that includes functions for the Weibull and lognormal distributions assuming the component is new. This tool is an extension of *Mathematica*. The new functions can be found in Appendix A. A palette for the *Mathematica* graphical user interface which provides buttons for the new functions can be found at Appendix B. Appendix C contains installation instructions for the new tool.

The new tool was used to model selected components. Chapters 2 and 3 apply the new tool to components whose reliability is modeled with the Weibull and lognormal distributions, respectively. Chapter 4 illustrates use of the new tool when analyzing and ranking replacement-before-failure rules for a collection of components. These chapters constitute a basic set of electronic templates for applying the new tool. A summary for the report is provided in Chapter 5.

The electronic form of each chapter and appendix of this report is a *Mathematica* 5 notebook. All of the methodology, computations and graphics in this report are *Mathematica* executables. The results were generated and inserted by *Mathematica*. Thus the technical content of this report is "live" in the sense that it can be re-executed as desired by readers working with the electronic version (provided they have a copy of *Mathematica*). Please refer to *The Mathematica Book* [Wolfram 1999] for information on this software. Additional information, including a free reader, is available at <http://www.wolfram.com/>.

# Chapter 2

## Conditional Weibull Distribution Example: Track Centerguide

---

### Introduction

This chapter illustrates the new Weibull distribution functions for components that are of any age. The illustration is with an actual component from a track subsystem that was subject to aging. This chapter may be used as an analysis template for components that age in accordance with the two-parameter Weibull distribution and are candidates for replacement before failure.

---

### Weibull Parameter Values for Track Centerguide

AMSAA analyzed test data from a track centerguide. The data were fit to a Weibull distribution. The shape and scale parameter estimates, respectively, were:

```
shapeCtrGuide = 5.14;  
scaleCtrGuide = 4602;
```

A shape parameter greater than one indicates that the component ages. A shape parameter of five or more indicates that strong aging is present. The estimates will be assumed to be the true values of the parameters.

*Mathematica* has built-in functions for the two-parameter Weibull distribution in the standard add-on package `Statistics`ContinuousDistributions``.

```
Needs["Statistics`ContinuousDistributions`"]
```

The usage message for the Weibull distribution is:

```
? WeibullDistribution
```

```
WeibullDistribution[alpha, beta] represents the Weibull  
distribution with shape parameter alpha and scale parameter beta. More...
```

The usage message for the cumulative distribution function, a very common distribution function, is:

## ? CDF

CDF[distribution, x] gives the cumulative distribution function of the specified statistical distribution evaluated at x. For continuous distributions, this is defined as the integral of the probability density function from the lowest value in the domain to x. For discrete distributions, this is defined as the sum of the probability density function from the lowest value in the domain to x.

Unfortunately, these functions are for the unconditional probability distributions, thereby assuming that the item is new. The new add-on package Reliability`ConditionalDistributions`, provided in Appendix A herein, contains more general functions for the two-parameter Weibull distribution where the components can be of any age. Before the new functions can be used, the add-on package Reliability`ConditionalDistributions` must be loaded:

```
Needs["Reliability`ConditionalDistributions`"]
```

The usage message for the current version of the package is:

## ? ConditionalDistributions

ConditionalDistributions.m (version 1.0) is a package that contains conditional distributions for the Weibull and lognormal distributions thereby supplementing many of the Weibull and lognormal functions in the standard add-on package Statistics`ContinuousDistributions.

The rest of this chapter will illustrate the use of these new functions with the example component. A list of the new functions is:

```
? Reliability`ConditionalDistributions`*
```

## Reliability`ConditionalDistributions`

ConditionalCDF

ConditionalMeanLife

ConditionalQuantile

ConditionalDistributions

ConditionalMeanLifeRemaining

ConditionalReliability

ConditionalHazard

ConditionalPDF

---

## Conditional CDF

One of the most useful functions is the conditional cumulative distribution function (CDF). The usage message is:

### ? ConditionalCDF

ConditionalCDF[distribution, t, tprime] gives the probability using the specified distribution that an item which has reached the age tprime will fail by time t.

A plot of the CDF curve for a new track centerguide can be generated from the ConditionalCDF function and the built-in function Plot:

```

plotnew = Plot[ConditionalCDF[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t, 0],
  {t, 0, 7000}, Axes → False, Frame → True,
  FrameLabel → {"t, miles", "Conditional Failure Probability",
    "Given Current Age 0 Miles", None}, PlotStyle → Hue[.0]];

```

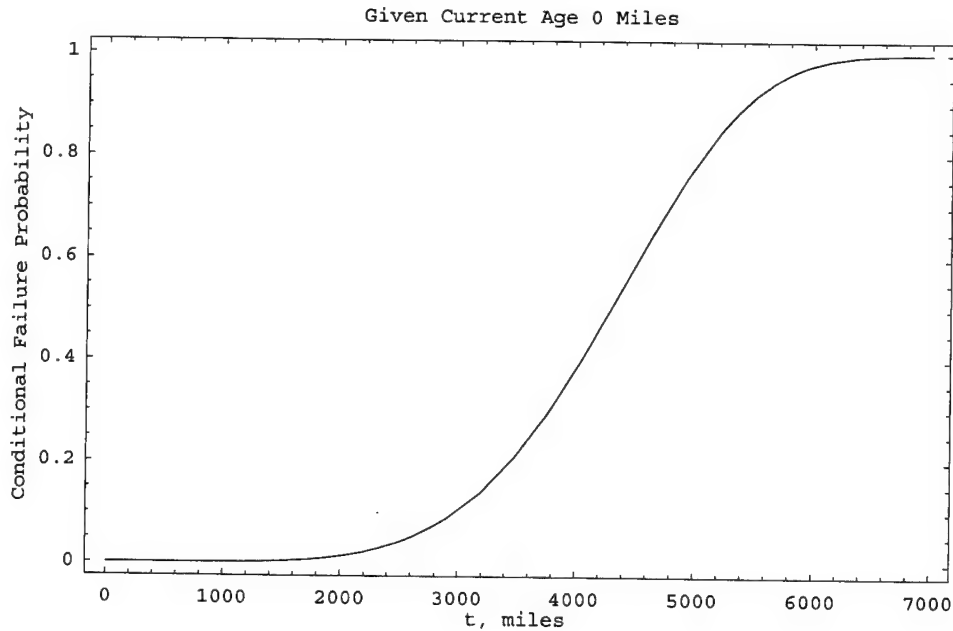


Figure 2-1

Examination of the curve reveals that there is little probability of failure before 3,000 miles and failure is quite likely to occur by 5,000 miles or so. A family of such curves with centerguides of various ages can be generated and plotted thus:

```

plot1000 =
  Plot[ConditionalCDF[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t, 1000],
    {t, 1000, 7000}, DisplayFunction → Identity, PlotStyle → Hue[.15]];

plot2000 =
  Plot[ConditionalCDF[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t, 2000],
    {t, 2000, 7000}, DisplayFunction → Identity, PlotStyle → Hue[.3]];

plot3000 =
  Plot[ConditionalCDF[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t, 3000],
    {t, 3000, 7000}, DisplayFunction → Identity, PlotStyle → Hue[.45]];

plot4000 =
  Plot[ConditionalCDF[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t, 4000],
    {t, 4000, 7000}, DisplayFunction → Identity, PlotStyle → Hue[.6]];

plot5000 =
  Plot[ConditionalCDF[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t, 5000],
    {t, 5000, 7000}, DisplayFunction → Identity, PlotStyle → Hue[.75]];

```

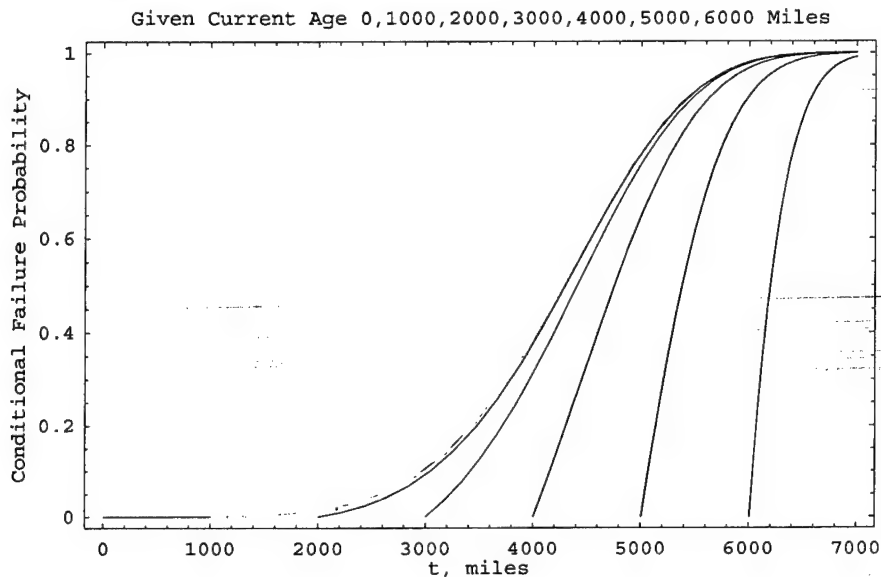


```

plot6000 =
  Plot[ConditionalCDF[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t, 6000],
    {t, 6000, 7000}, DisplayFunction->Identity, PlotStyle->Hue[.9]];

Show[plotnew, plot1000, plot2000, plot3000, plot4000, plot5000,
  plot6000, FrameLabel->{"t, miles", "Conditional Failure Probability",
    "Given Current Age 0,1000,2000,3000,4000,5000,6000 Miles", None},
  DisplayFunction->$DisplayFunction];

```



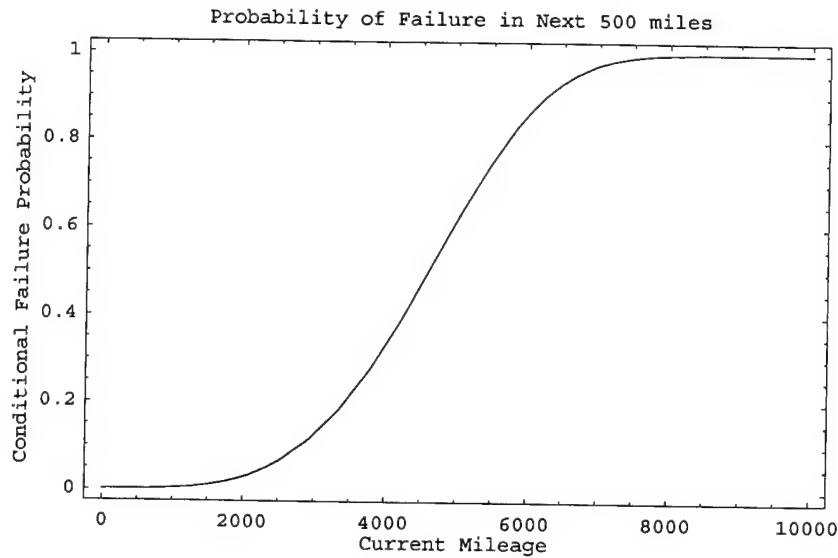
**Figure 2-2**

The plot above shows that centerguides that haven't failed by 3,000 are quite likely to fail by 5,000 miles or so. Centerguides that haven't failed by 5,000 miles are quite likely to fail in the next 500 miles.

The situation may arise where it will be rather inconvenient for the component to fail during the next 500 miles, perhaps. This situation may arise because the system is to be deployed and is expected to undergo 500 miles of usage before a maintenance pulse will occur. We can use `ConditionalCDF` to plot the probability of failing in next 500 miles as a function of component age thus:

```
Plot[ConditionalCDF[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], age + 500, age],
{age, 0, 10000}, Axes → False, Frame → True,
FrameLabel → {"Current Mileage", "Conditional Failure Probability",
"Probability of Failure in Next 500 miles", None}, PlotStyle → RGBColor[0, 0, 1]];

```



**Figure 2-3**

The plot shows that when the current age reaches approximately 4,500 miles, there is a 50% chance of it failing during the next 500 miles. Above 4,500 miles, the likelihood of failure is even greater. A table of these values is generated next:

```
TableForm[Table[{age, ConditionalCDF[WeibullDistribution[shapeCtrGuide, scaleCtrGuide],
age + 500, age]}, {age, 0, 10000, 500}],
TableHeadings -> {None, {"Age(miles)", "CDF(next 500)"}}, TableAlignments -> Center]
```

Age(miles)	CDF(next 500)
0	0.0000110958
500	0.000380082
1000	0.00274955
1500	0.0105947
2000	0.0292068
2500	0.0652195
3000	0.125414
3500	0.214605
4000	0.332841
4500	0.472955
5000	0.620243
5500	0.755832
6000	0.863293
6500	0.935121
7000	0.974661
7500	0.992113
8000	0.998111
8500	0.999664
9000	0.999958
9500	0.999996
10000	1.

Perhaps all centerguides that have 4,500 or more miles should be replaced prior to deployment.

---

## Conditional Quantile

The conditional quantile function is the inverse of the CDF. The usage message is:

### ? ConditionalQuantile

```
ConditionalQuantile[distribution, tprime, q] gives the qth quantile
using the specified distribution for an item that has survived to age tprime.
```

For example, if we want to know at what age 50% of the centerguides that have already survived 3,500 miles are likely to fail, we can use the conditional quantile function:

```
ConditionalQuantile[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], 3500, .5]

4545.08
```

This is often referred to as the median. This quantile just obtained can be plugged back into the conditional CDF and we should obtain a probability of 0.5:

```
ConditionalCDF[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], %, 3500]

0.5
```

And we do. In contrast, the median life of a new centerguide is:

```
ConditionalQuantile[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], 0, .5]  
4285.28
```

So there is a 50% chance that a new centerguide will survive until 4,285 miles whereas a centerguide that has survived until 3,500 miles has a 50% chance of surviving to 4,545 miles.

It appears that the conditional quantile function may be helpful when considering the replacement of centerguides that are already in service before failure.

---

## Conditional Reliability

The next function to be examined is the conditional reliability function. The reliability function is one minus the CDF. The usage message is:

```
? ConditionalReliability
```

```
ConditionalReliability[distribution, t, tprime] gives the probability using the specified  
distribution that an item which has reached the age tprime will survive to time t.
```

A plot that generates a family of reliability curves for centerguides of various ages, essentially the complement of figure 2-2, is generated thus:

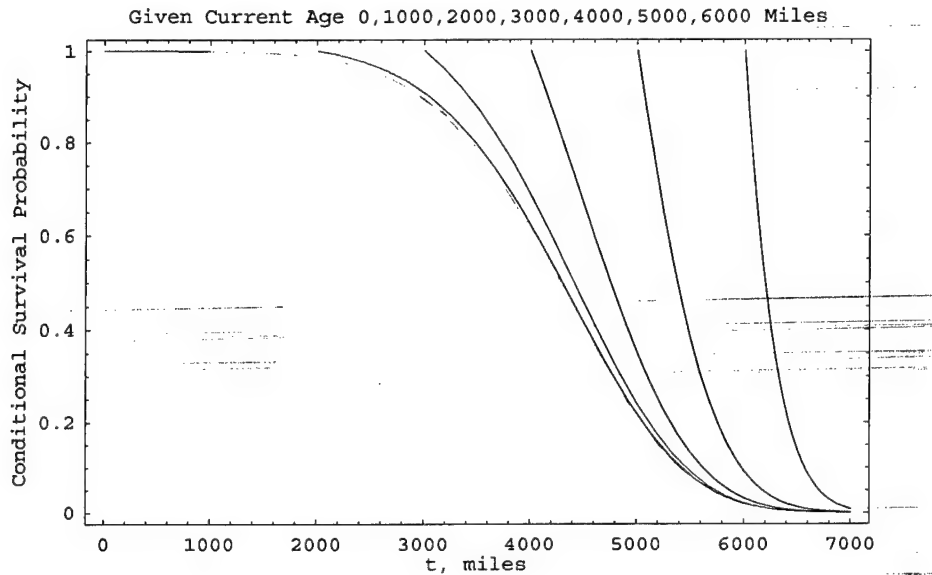
```
plotnew =  
  Plot[ConditionalReliability[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t, 0],  
    {t, 0, 7000}, DisplayFunction -> Identity, PlotStyle -> Hue[.0]];  
  
plot1000 = Plot[  
  ConditionalReliability[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t, 1000],  
  {t, 1000, 7000}, DisplayFunction -> Identity, PlotStyle -> Hue[.15]];  
  
plot2000 = Plot[  
  ConditionalReliability[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t, 2000],  
  {t, 2000, 7000}, DisplayFunction -> Identity, PlotStyle -> Hue[.3]];  
  
plot3000 = Plot[  
  ConditionalReliability[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t, 3000],  
  {t, 3000, 7000}, DisplayFunction -> Identity, PlotStyle -> Hue[.45]];  
  
plot4000 = Plot[  
  ConditionalReliability[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t, 4000],  
  {t, 4000, 7000}, DisplayFunction -> Identity, PlotStyle -> Hue[.6]];  
  
plot5000 = Plot[  
  ConditionalReliability[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t, 5000],  
  {t, 5000, 7000}, DisplayFunction -> Identity, PlotStyle -> Hue[.75]];
```

```

plot6000 = Plot[
  ConditionalReliability[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t, 6000],
  {t, 6000, 7000}, DisplayFunction -> Identity, PlotStyle -> Hue[.9]];

Show[plotnew, plot1000, plot2000, plot3000, plot4000, plot5000, plot6000, Axes -> False,
  Frame -> True, FrameLabel -> {"t, miles", "Conditional Survival Probability",
    "Given Current Age 0,1000,2000,3000,4000,5000,6000 Miles", None},
  DisplayFunction -> $DisplayFunction];

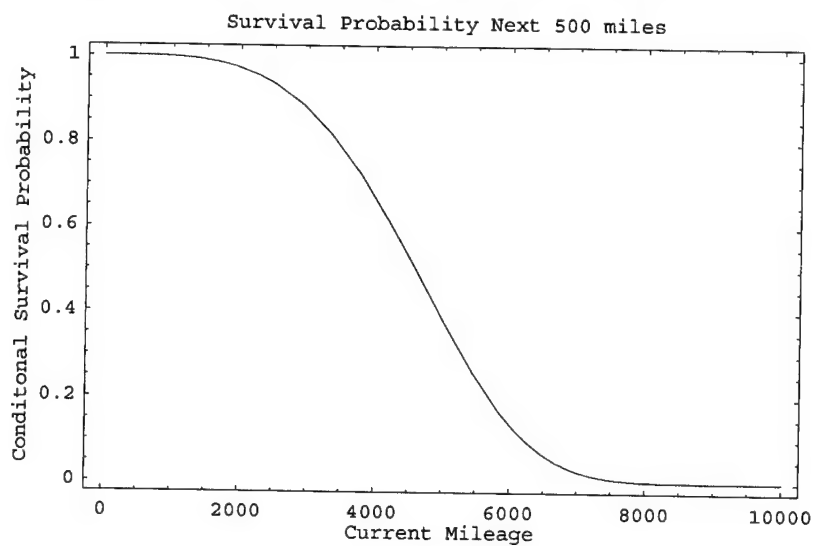
```



**Figure 2-4**

From the figure above one can see that a new centerguide is highly reliable for a few thousand miles whereas those that survive to several thousand miles or so are quite unreliable. We now plot the probability of surviving the next 500 miles as a function of age:

```
Plot[ConditionalReliability[WeibullDistribution[shapeCtrGuide, scaleCtrGuide],
  age + 500, age], {age, 0, 10000}, Axes → False, Frame → True,
  FrameLabel → {"Current Mileage", "Conditonal Survival Probability",
    "Survival Probability Next 500 miles", None}, PlotStyle → RGBColor[0, 1, 0]];
```



**Figure 2-5**

Centerguides that have survived to approximately 4,500 miles, have a 50% chance of failing in the next 500 miles. A table of such values is generated thus:

```
TableForm[
  Table[{age, ConditionalReliability[WeibullDistribution[shapeCtrGuide, scaleCtrGuide],
    age+500, age]}, {age, 0, 10000, 500}],
  TableHeadings -> {None, {"Age(miles)", "R(next 500)"}}, TableAlignments -> Center]
```

Age(miles)	R(next 500)
0	0.999989
500	0.99962
1000	0.99725
1500	0.989405
2000	0.970793
2500	0.934781
3000	0.874586
3500	0.785395
4000	0.667159
4500	0.527045
5000	0.379757
5500	0.244168
6000	0.136707
6500	0.064879
7000	0.0253393
7500	0.00788686
8000	0.00188943
8500	0.000335596
9000	0.0000424567
9500	$3.66546 \times 10^{-6}$
10000	$2.06343 \times 10^{-7}$

A table with both the conditional CDF and reliability values is generated as follows:

```
TableForm[
  Table[{age, ConditionalCDF[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], age + 500,
    age], ConditionalReliability[WeibullDistribution[shapeCtrGuide, scaleCtrGuide],
    age + 500, age]}, {age, 0, 10000, 500}],
  TableHeadings → {None, {"Age(miles)", "CDF(next 500)", "R(next 500)"}},
  TableAlignments → Center]
```

Age(miles)	CDF(next 500)	R(next 500)
0	0.0000110958	0.999989
500	0.000380082	0.99962
1000	0.00274955	0.99725
1500	0.0105947	0.989405
2000	0.0292068	0.970793
2500	0.0652195	0.934781
3000	0.125414	0.874586
3500	0.214605	0.785395
4000	0.332841	0.667159
4500	0.472955	0.527045
5000	0.620243	0.379757
5500	0.755832	0.244168
6000	0.863293	0.136707
6500	0.935121	0.064879
7000	0.974661	0.0253393
7500	0.992113	0.00788686
8000	0.998111	0.00188943
8500	0.999664	0.000335596
9000	0.999958	0.0000424567
9500	0.999996	$3.66546 \times 10^{-6}$
10000	1.	$2.06343 \times 10^{-7}$

---

## Conditional PDF

Next the conditional probability density function (PDF) will be considered. Its usage message is:

### ? ConditionalPDF

ConditionalPDF[distribution, t, tprime] gives the probability density function evaluated at t for an item which has reached the age tprime using the specified distribution.

A plot that generates a family of PDF curves for centerguides of various ages is generated thus:

```
plotnew = Plot[ConditionalPDF[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t, 0],
  {t, 0, 7000}, DisplayFunction → Identity, PlotStyle → Hue[.0]];

plot1000 =
  Plot[ConditionalPDF[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t, 1000],
  {t, 1000, 7000}, DisplayFunction → Identity, PlotStyle → Hue[.15]];

plot2000 =
  Plot[ConditionalPDF[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t, 2000],
  {t, 2000, 7000}, DisplayFunction → Identity, PlotStyle → Hue[.3]];
```



```

plot3000 =
  Plot[ConditionalPDF[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t, 3000],
    {t, 3000, 7000}, DisplayFunction->Identity, PlotStyle->Hue[.45]];

plot4000 =
  Plot[ConditionalPDF[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t, 4000],
    {t, 4000, 7000}, DisplayFunction->Identity, PlotStyle->Hue[.6]];

plot5000 =
  Plot[ConditionalPDF[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t, 5000],
    {t, 5000, 7000}, DisplayFunction->Identity, PlotStyle->Hue[.75]];

plot6000 =
  Plot[ConditionalPDF[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t, 6000],
    {t, 6000, 7000}, DisplayFunction->Identity, PlotStyle->Hue[.9]];

Show[plotnew, plot1000, plot2000, plot3000, plot4000, plot5000, plot6000,
  Axes->False, Frame->True, FrameLabel->{"t, miles", "Conditional PDF",
    "Given Current Age 0,1000,2000,3000,4000,5000,6000 Miles", None},
  DisplayFunction->$DisplayFunction];

```



Figure 2-6

The curve that starts at zero on the mileage axis corresponds to a new centerguide. The further to the right a curve starts, the older the centerguide is. Figure 2-6 shows that centerguides that are older have increased probability density. The PDF is essentially re-scaled by component age so that the area beneath it equals one. It is difficult, however, to obtain any specific quantitative insight concerning when to replace centerguides of various ages.

---

## Conditional Hazard

The next new function that we will consider here is the conditional hazard or failure-rate function. The hazard function is frequently quite useful for component reliability analysis. The usage message is:

**? ConditionalHazard**

ConditionalHazard[distribution, t] gives the hazard function evaluated at t for an item using the specified distribution. The conditional hazard is unaffected by the age of the item

The conditional hazard function is the same as the unconditional. Let us graph these functions in order to visualize this more clearly.

```
plotnew = Plot[ConditionalHazard[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t],  
  {t, 0, 7000}, DisplayFunction -> Identity, PlotStyle -> Hue[.0]];  
  
plot1000 = Plot[ConditionalHazard[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t],  
  {t, 1000, 7000}, DisplayFunction -> Identity, PlotStyle -> Hue[.15]];  
  
plot2000 = Plot[ConditionalHazard[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t],  
  {t, 2000, 7000}, DisplayFunction -> Identity, PlotStyle -> Hue[.3]];  
  
plot3000 = Plot[ConditionalHazard[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t],  
  {t, 3000, 7000}, DisplayFunction -> Identity, PlotStyle -> Hue[.45]];  
  
plot4000 = Plot[ConditionalHazard[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t],  
  {t, 4000, 7000}, DisplayFunction -> Identity, PlotStyle -> Hue[.6]];  
  
plot5000 = Plot[ConditionalHazard[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t],  
  {t, 5000, 7000}, DisplayFunction -> Identity, PlotStyle -> Hue[.75]];  
  
plot6000 = Plot[ConditionalHazard[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t],  
  {t, 6000, 7000}, DisplayFunction -> Identity, PlotStyle -> Hue[.9]];
```

```
Show[plotnew, plot1000, plot2000, plot3000, plot4000, plot5000, plot6000,
  Axes → False, Frame → True, FrameLabel → {"t, miles", "Conditional Hazard",
    "Given Current Age 0,1000,2000,3000,4000,5000,6000 Miles", None},
  DisplayFunction → $DisplayFunction];
```

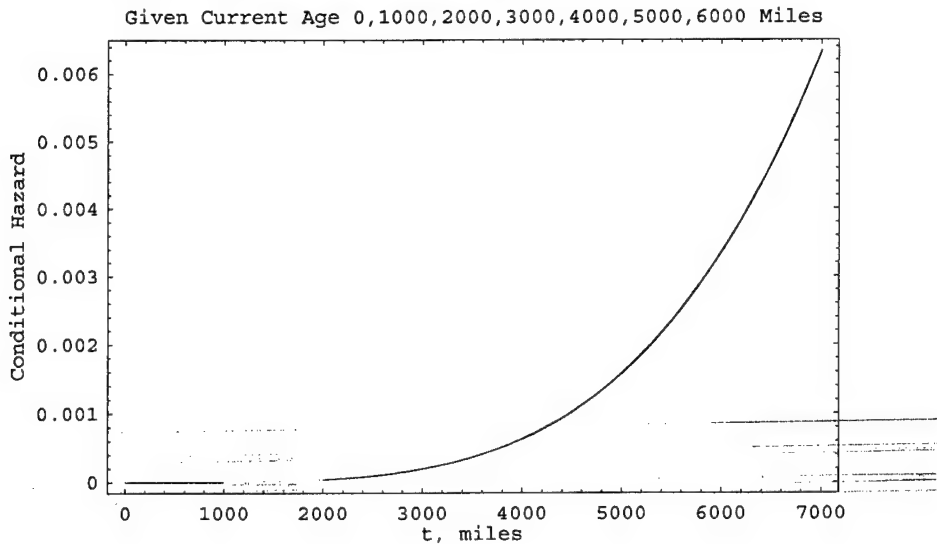


Figure 2-7

The age of the component does not change the hazard function but it does remove the portion of the function to the left of the component age. This is why the color of the curve changes every 1,000 miles. The hazard curve increases steadily which indicates aging. The hazard function can be viewed as the probability of failing in the next interval of time given the component has survived up to that point in time. A new component faces little hazard for the first few thousand miles. In contrast, the hazard curve for centerguide that has survived to 6,000 miles is quite steep.

The hazard function might be useful for evaluating when to replace components of various ages.

---

## Conditional Mean Life

The next function to be considered is the conditional mean life. The usage message is:

```
? ConditionalMeanLife
```

```
ConditionalMeanLife[distribution, tprime] gives the conditional mean age at failure
for an item which has reached the age tprime using the specified distribution.
```

We can generate a graph of the conditional mean life of the track centerguide as a function of its age thus:

```
Plot[ConditionalMeanLife[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], age],
{age, 0, 7000}, PlotRange → All, Axes → False, Frame → True,
FrameLabel → {"age,miles", "Mean Life", StringForm["Weibull Shape = `", Scale = `",
shapeCtrGuide, scaleCtrGuide], None}, PlotStyle → RGBColor[.5, 0, .5]];
```

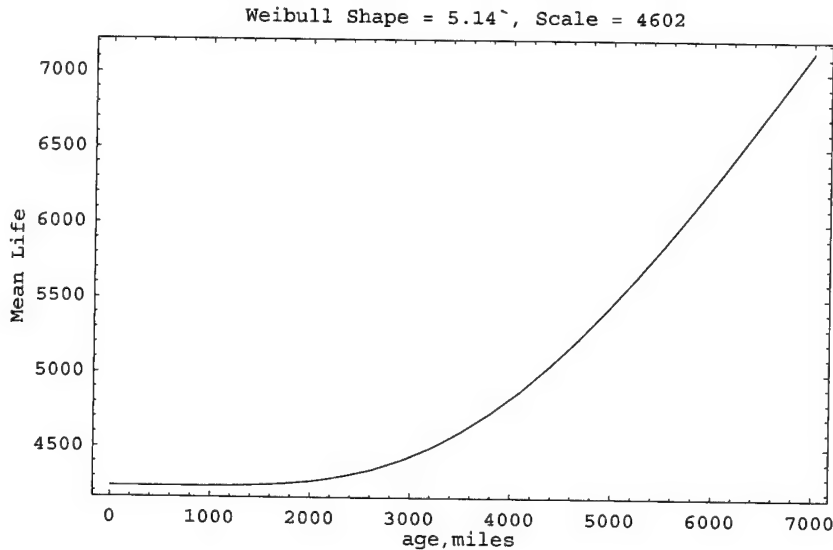


Figure 2-8

One insight from the graph above is that once a centerguide has successfully reached an age of several thousand miles, one shouldn't expect it to last much longer. Next, let us tabulate values of centerguide mean life.

```
TableForm[Table[
{age, ConditionalMeanLife[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], age]},
{age, 0, 7000, 500}],
TableHeadings → {None, {"Age(miles)", "Mean Life (miles)"}, TableAlignments → Center]
```

Age(miles)	Mean Life (miles)
0	4232.15
500	4232.19
1000	4233.48
1500	4241.53
2000	4267.7
2500	4327.3
3000	4435.38
3500	4602.38
4000	4831.77
4500	5120.29
5000	5460.07
5500	5841.34
6000	6254.52
6500	6691.36
7000	7145.37

What one may infer from the table and the graph is that once a centerguide has reached 5,000 miles, it should not be expected to survive another 500 miles. This may be very helpful when considering when to replace centerguides of various ages.

While the mean life function was interesting, the mean life remaining function may offer additional insight.

---

## Conditional Mean Life Remaining

The final new function to be considered is the conditional mean life remaining function, the usage message for which is:

```
? ConditionalMeanLifeRemaining
```

```
ConditionalMeanLifeRemaining[distribution, tprime] gives the conditional mean life remaining at failure for an item which has reached the age tprime using the specified distribution.
```

We can plot the mean life remaining for the centerguide thus:

```
Plot[ConditionalMeanLifeRemaining[
  WeibullDistribution[shapeCtrGuide, scaleCtrGuide], age], {age, 0, 7000},
  Axes → False, Frame → True, FrameLabel → {"age,miles", "Mean Life Remaining",
  StringForm["Weibull Shape = `", Scale = `", shapeCtrGuide, scaleCtrGuide],
  None}, PlotStyle → RGBColor[.5, 0, .5]];
```

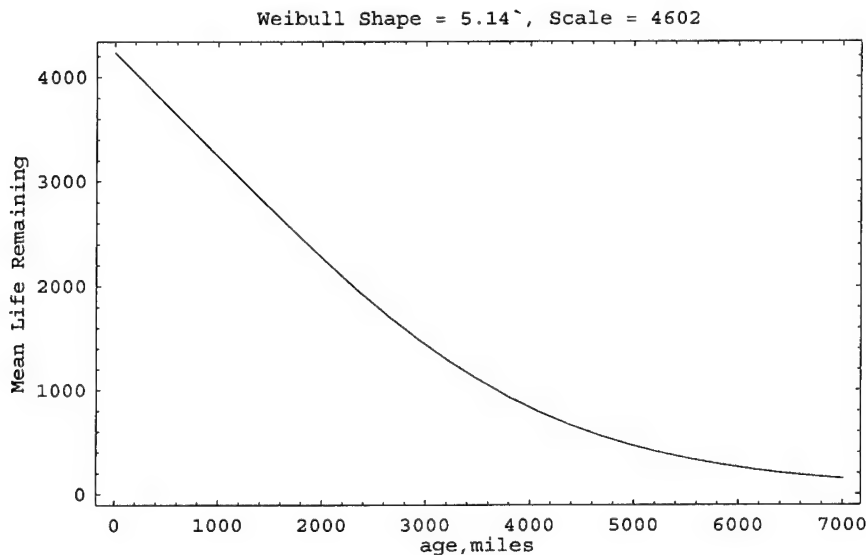


Figure 2-9

Examination of the graph above reveals that if a centerguide survives to several thousand miles, little additional life should be expected. This is the same insight as we obtained from figure 2-8 but it more readily seen.

Tabulated mean life remaining values are generated as follows:

```
TableForm[Table[{age, ConditionalMeanLifeRemaining[
  WeibullDistribution[shapeCtrGuide, scaleCtrGuide], age]], {age, 0, 7000, 500}],
TableHeadings -> {None, {"Age(miles)", "Mean Life Remaining(miles)"}},
TableAlignments -> Center]
```

Age(miles)	Mean Life Remaining(miles)
0	4232.15
500	3732.19
1000	3233.48
1500	2741.53
2000	2267.7
2500	1827.3
3000	1435.38
3500	1102.38
4000	831.772
4500	620.286
5000	460.07
5500	341.344
6000	254.516
6500	191.356
7000	145.371

Perhaps it would be helpful to generate a table with columns for mean life remaining and percentage of mean life remaining compared to the mean life of a new centerguide:

```
TableForm[Table[
  {age, ConditionalMeanLifeRemaining[WeibullDistribution[shapeCtrGuide, scaleCtrGuide],
    age], 100 * (ConditionalMeanLifeRemaining[
      WeibullDistribution[shapeCtrGuide, scaleCtrGuide], age]) /
    ConditionalMeanLife[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], 0]},
  {age, 0, 7000, 500}], TableHeadings -> {None, {"Age(miles)", "Mean Life Remaining",
    "% Mean Life Remaining (New)"}}, TableAlignments -> Center]
```

Age(miles)	Mean Life Remaining	% Mean Life Remaining (New)
0	4232.15	100.
500	3732.19	88.1867
1000	3233.48	76.4027
1500	2741.53	64.7785
2000	2267.7	53.5826
2500	1827.3	43.1767
3000	1435.38	33.9162
3500	1102.38	26.0478
4000	831.772	19.6536
4500	620.286	14.6565
5000	460.07	10.8708
5500	341.344	8.0655
6000	254.516	6.01387
6500	191.356	4.52147
7000	145.371	3.43493

It may be more appropriate to replace the percentage of mean life remaining when compared to the mean life of a component of the same age:

```
TableForm[Table[
  {age, ConditionalMeanLifeRemaining[WeibullDistribution[shapeCtrGuide, scaleCtrGuide],
    age], 100 * (ConditionalMeanLifeRemaining[
      WeibullDistribution[shapeCtrGuide, scaleCtrGuide], age]) /
    ConditionalMeanLife[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], age]},
  {age, 0, 7000, 500}], TableHeadings -> {None, {"Age(miles)", "Mean Life Remaining",
    "% Mean Life Remaining (Used)"}}, TableAlignments -> Center]
```

Age(miles)	Mean Life Remaining	% Mean Life Remaining (Used)
0	4232.15	100.
500	3732.19	88.1858
1000	3233.48	76.3788
1500	2741.53	64.6354
2000	2267.7	53.1363
2500	1827.3	42.2273
3000	1435.38	32.3621
3500	1102.38	23.9524
4000	831.772	17.2146
4500	620.286	12.1143
5000	460.07	8.42608
5500	341.344	5.84359
6000	254.516	4.06932
6500	191.356	2.85974
7000	145.371	2.03448

The percentage of mean life remaining column was changed very little. Finally, we can include columns for the conditional CDF and reliability that appeared earlier in this chapter to the table above:

```
TableForm[
  Table[{age, ConditionalCDF[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], age + 500,
    age], ConditionalReliability[WeibullDistribution[shapeCtrGuide, scaleCtrGuide],
    age + 500, age], ConditionalMeanLifeRemaining[WeibullDistribution[
    shapeCtrGuide, scaleCtrGuide], age], 100 * (ConditionalMeanLifeRemaining[
    WeibullDistribution[shapeCtrGuide, scaleCtrGuide], age]) /
    ConditionalMeanLife[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], age]}],
  {age, 0, 7000, 500}], TableHeadings -> {None, {"Age(miles)", "CDF(next 500)",
  "R(next 500)", "Mean Life Rem.", "% Mean Life Rem. (Used)"}},
  TableAlignments -> Center, TableSpacing -> {1, 1.5}]
```

Age(miles)	CDF(next 500)	R(next 500)	Mean Life Rem.	% Mean Life Rem. (Used)
0	0.0000110958	0.999989	4232.15	100.
500	0.000380082	0.99962	3732.19	88.1858
1000	0.00274955	0.99725	3233.48	76.3788
1500	0.0105947	0.989405	2741.53	64.6354
2000	0.0292068	0.970793	2267.7	53.1363
2500	0.0652195	0.934781	1827.3	42.2273
3000	0.125414	0.874586	1435.38	32.3621
3500	0.214605	0.785395	1102.38	23.9524
4000	0.332841	0.667159	831.772	17.2146
4500	0.472955	0.527045	620.286	12.1143
5000	0.620243	0.379757	460.07	8.42608
5500	0.755832	0.244168	341.344	5.84359
6000	0.863293	0.136707	254.516	4.06932
6500	0.935121	0.064879	191.356	2.85974
7000	0.974661	0.0253393	145.371	2.03448

Examination of the metrics in the table above reveals that at around 4,500 miles, there's a 50% chance of a failure in the next 500 miles, at that point the centerguide should be expected to last another 620 miles or so and this is only 12% of the mean life of a centerguide of that age. One should be able to develop an interactive decision-making process for component replacement before failure based on this collection of metrics.

---

## Summary

In this chapter we set out to illustrate the new conditional distributions for the two-parameter Weibull distribution. We discovered that it is difficult to obtain much insight from a PDF graph with respect to the replacement of centerguides of various ages before failure. All of the other new functions appear to be valuable for evaluating the replacement of aging components before failure. These functions generate metrics, such as those in the last table above, that provide a reasonable foundation for the development of an interactive decision-making process for component replacement before failure.



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# Chapter 3

## Conditional Lognormal Distribution: Shock Absorber Example

---

### Introduction

This chapter illustrates the new lognormal distribution functions for components that are of any age. The component used in this chapter is a shock absorber, the data and analysis for which was found in the literature. This chapter may be used as an analysis template for components that age in accordance with the two-parameter lognormal distribution and are candidates for replacement before failure.

---

### Lognormal Parameter Values for Shock Absorber

Meeker and Escobar (1998, Table 8.1) provide estimates for the lognormal distribution for the shock absorber data reported in O'Connor (1985, p. 85). The log (base  $e$ ) mean and standard deviation estimates, respectively, are:

```
muSA = 10.14;
```

```
sigmaSA = .5301;
```

These estimates will be assumed to be the true values of the parameters.

*Mathematica* has built-in functions for the two-parameter lognormal distribution in the standard add-on package `Statistics`ContinuousDistributions``.

```
Needs["Statistics`ContinuousDistributions`"]
```

The usage message for the lognormal distribution is:

```
? LogNormalDistribution
```

```
LogNormalDistribution[mu, sigma] represents the log-normal distribution based  
on a normal distribution having mean mu and standard deviation sigma. More...
```

As discussed in the previous chapter, these functions are for the unconditional probability distributions, thereby assuming that the item is new. The new add-on package `Reliability`ConditionalDistributions``, provided in Appendix A herein, contains more general functions for the two-parameter lognormal distribution where the components can be of any age. Before the new functions can be used, the add-on package `ConditionalDistributions` must be loaded:

```
Needs["Reliability`ConditionalDistributions`"]
```

The rest of this chapter will illustrate the use of these new functions with the example component.

---

## Conditional CDF

A plot of the CDF curve for a new shock absorber can be generated from the ConditionalCDF function and the built-in function Plot:

```
plotnew = Plot[ConditionalCDF[LogNormalDistribution[muSA, sigmaSA], t, 0],  
  {t, 0, 70000}, Axes → False, Frame → True, FrameLabel →  
  {"t, miles", "Conditional Failure Probability", "Given Current Age 0 Miles", None},  
  PlotStyle → Hue[.0], PlotRange → All];
```

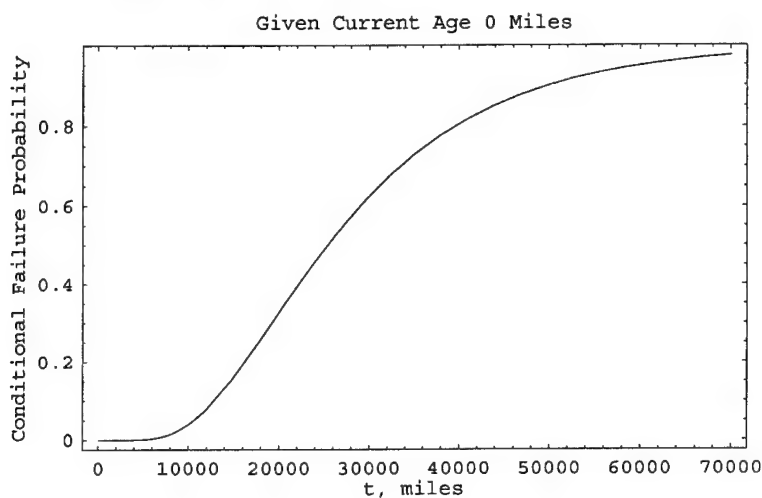


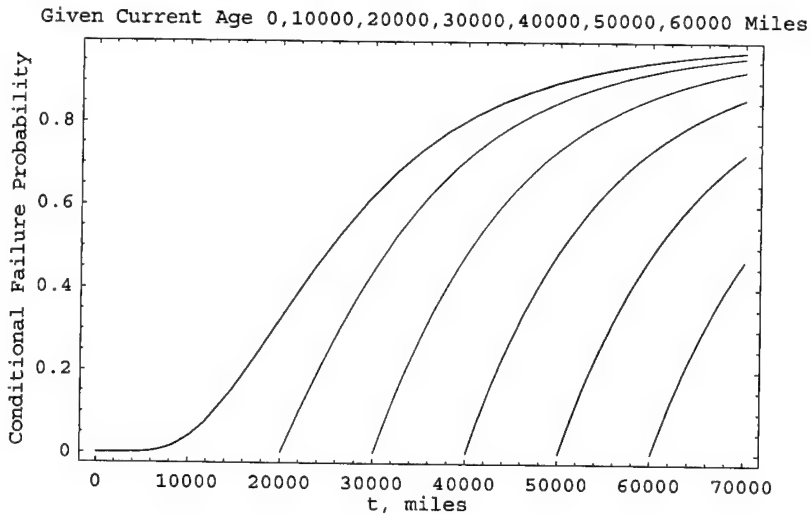
Figure 3-1

Examination of the curve reveals that there is little probability of failure before 10,000 miles and failure is quite likely to occur by 40,000 miles or so. A family of such curves with shock absorbers of various ages can be generated and plotted thus:

```
plot10000 = Plot[ConditionalCDF[LogNormalDistribution[muSA, sigmaSA], t, 10000],  
  {t, 10000, 70000}, DisplayFunction → Identity, PlotStyle → Hue[.15]];  
  
plot20000 = Plot[ConditionalCDF[LogNormalDistribution[muSA, sigmaSA], t, 20000],  
  {t, 20000, 70000}, DisplayFunction → Identity, PlotStyle → Hue[.3]];  
  
plot30000 = Plot[ConditionalCDF[LogNormalDistribution[muSA, sigmaSA], t, 30000],  
  {t, 30000, 70000}, DisplayFunction → Identity, PlotStyle → Hue[.45]];  
  
plot40000 = Plot[ConditionalCDF[LogNormalDistribution[muSA, sigmaSA], t, 40000],  
  {t, 40000, 70000}, DisplayFunction → Identity, PlotStyle → Hue[.6]];  
  
plot50000 = Plot[ConditionalCDF[LogNormalDistribution[muSA, sigmaSA], t, 50000],  
  {t, 50000, 70000}, DisplayFunction → Identity, PlotStyle → Hue[.75]];
```

```
plot60000 = Plot[ConditionalCDF[LogNormalDistribution[muSA, sigmaSA], t, 60000],
  {t, 60000, 70000}, DisplayFunction->Identity, PlotStyle->Hue[.9]];
```

```
Show[plotnew, plot10000, plot20000, plot30000, plot40000, plot50000,
  plot60000, FrameLabel->{"t, miles", "Conditional Failure Probability",
    "Given Current Age 0,10000,20000,30000,40000,50000,60000 Miles", None},
  DisplayFunction->$DisplayFunction];
```



**Figure 3-2**

The plot above shows that shock absorbers are unlikely to fail by 10,000 miles and the accumulate risk very gradually thereafter.

The situation may arise where it will be rather inconvenient for the component to fail during the next 2,000 miles, perhaps. This situation may arise because the system is to be deployed and is expected to undergo 2,000 miles of usage before a maintenance pulse will occur. We can use ConditionalCDF to plot the probability of failing in next 2,000 miles as a function of component age thus:

```
Plot[ConditionalCDF[LogNormalDistribution[muSA, sigmaSA], age + 2000, age],
{age, 0, 70000}, Axes → False, Frame → True,
FrameLabel → {"Current Mileage", "Conditional Failure Probability",
"Probability of Failure in Next 2000 miles", None}, PlotStyle → RGBColor[0, 0, 1]];
```

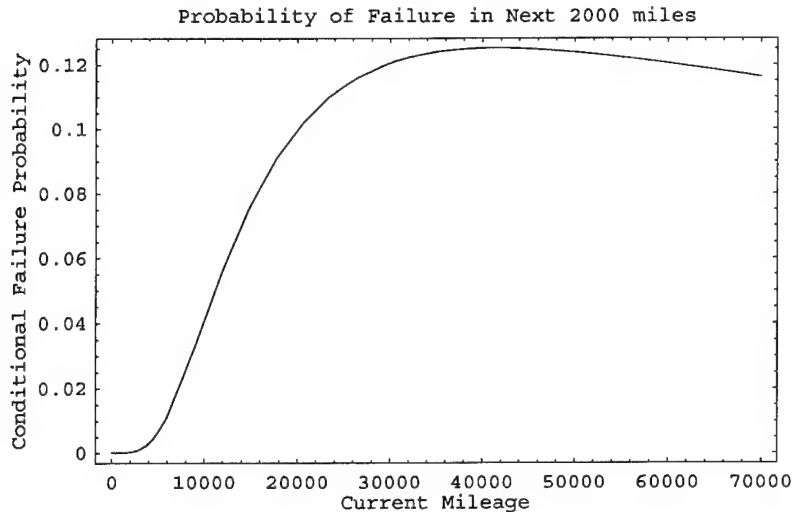


Figure 3-3

The plot above declines after 40,000 miles which is unrealistic. This is an unfortunate behavior of the lognormal distribution, that will be seen in the hazard function section of this chapter. We can generate a table of these values thus:

```
TableForm[
Table[{age, ConditionalCDF[LogNormalDistribution[muSA, sigmaSA], age + 2000, age]},
{age, 0, 70000, 5000}],
TableHeadings → {None, {"Age(miles)", "CDF(next 2000)"}, TableAlignments → Center]
```

Age(miles)	CDF(next 2000)
0	$8.34545 \times 10^{-7}$
5000	0.00652615
10000	0.0411984
15000	0.0768332
20000	0.100019
25000	0.113327
30000	0.120464
35000	0.123885
40000	0.12506
45000	0.124861
50000	0.123814
55000	0.122248
60000	0.120368
65000	0.118308
70000	0.116154

There does not appear to be an obvious choice as to when shock absorbers should be replaced.

---

## Conditional Hazard

The next function that we will illustrate here is the conditional hazard or failure-rate function.

```
plotnew = Plot[ConditionalHazard[LogNormalDistribution[muSA, sigmaSA], t],  
  {t, 0, 70000}, DisplayFunction -> Identity, PlotStyle -> Hue[.0]];  
  
plot10000 = Plot[ConditionalHazard[LogNormalDistribution[muSA, sigmaSA], t],  
  {t, 10000, 70000}, DisplayFunction -> Identity, PlotStyle -> Hue[.15]];  
  
plot20000 = Plot[ConditionalHazard[LogNormalDistribution[muSA, sigmaSA], t],  
  {t, 20000, 70000}, DisplayFunction -> Identity, PlotStyle -> Hue[.3]];  
  
plot30000 = Plot[ConditionalHazard[LogNormalDistribution[muSA, sigmaSA], t],  
  {t, 30000, 70000}, DisplayFunction -> Identity, PlotStyle -> Hue[.45]];  
  
plot40000 = Plot[ConditionalHazard[LogNormalDistribution[muSA, sigmaSA], t],  
  {t, 40000, 70000}, DisplayFunction -> Identity, PlotStyle -> Hue[.6]];  
  
plot50000 = Plot[ConditionalHazard[LogNormalDistribution[muSA, sigmaSA], t],  
  {t, 50000, 70000}, DisplayFunction -> Identity, PlotStyle -> Hue[.75]];  
  
plot60000 = Plot[ConditionalHazard[LogNormalDistribution[muSA, sigmaSA], t],  
  {t, 60000, 70000}, DisplayFunction -> Identity, PlotStyle -> Hue[.9]];  
  
Show[plotnew, plot10000, plot20000, plot30000, plot40000, plot50000, plot60000,  
  Axes -> False, Frame -> True, FrameLabel -> {"t, miles", "Conditional Hazard",  
    "Given Current Age 0,10000,20000,30000,40000,50000,60000 Miles", None},  
  DisplayFunction -> $DisplayFunction, PlotRange -> All];
```

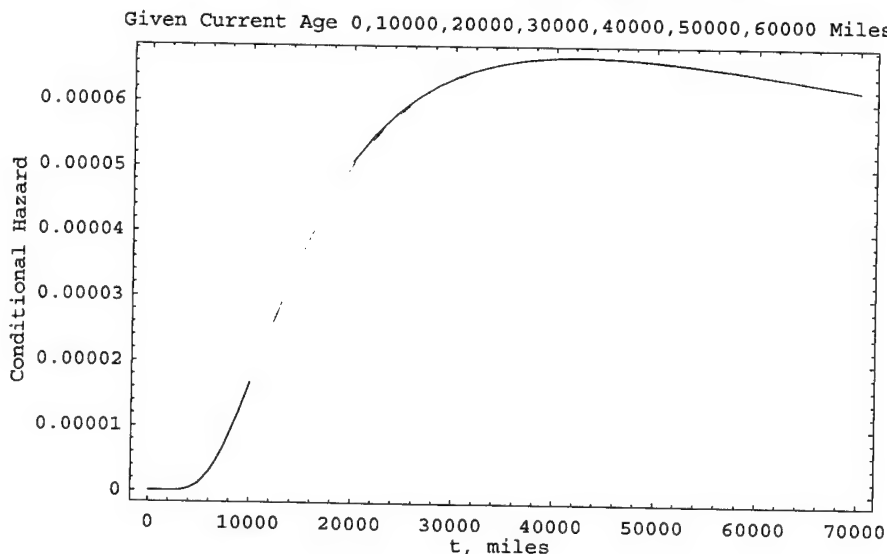


Figure 3-4

The age of the component does not change the hazard function but it does remove the portion of the function to the left of the component age. This is why the color of the curve changes every 10,000 miles. Until approximately 30,000 miles, the hazard curve increases steadily which indicates aging. The hazard function can be viewed as the probability of failing in the next interval of time given the component has survived up to that point in time. A new component faces little hazard for the first several thousand miles. In contrast, the hazard curve for centerguide that has survived to 10,000 miles is quite steep.

The hazard function might be useful for evaluating when to replace components of various ages.

---

## Conditional Quantile

The conditional quantile function is the inverse of the CDF. For example, if we want to know at what age 50% of the shock absorbers that have already survived 10,000 miles are likely to fail, we can use the conditional quantile function:

```
ConditionalQuantile[LogNormalDistribution[muSA, sigmaSA], 40000, .5]
50399.3
```

This is often referred to as the median. This quantile just obtained can be plugged back into the conditional CDF and we should obtain a probability of 0.5:

```
ConditionalCDF[LogNormalDistribution[muSA, sigmaSA], %, 40000]
0.5
```

And we do. In contrast, the median life of a new shock absorber is:

```
ConditionalQuantile[LogNormalDistribution[muSA, sigmaSA], 0, .5]
25336.5
```

So there is a 50% chance that a new shock absorber will survive until 25,337 miles whereas a centerguide that has survived until 40,000 miles has a 50% chance of surviving to 50,399 miles.

It appears that the conditional quantile function may be helpful when considering the replacement of shock absorbers that are already in service before failure.

---

## Conditional Reliability

The next function to be examined is the conditional reliability function. The reliability function is one minus the CDF. A plot that generates a family of reliability curves for centerguides of various ages, essentially the complement of figure 3-2, is generated thus:

```
plotnew = Plot[ConditionalReliability[LogNormalDistribution[muSA, sigmaSA], t, 0],
{t, 0, 70000}, DisplayFunction -> Identity, PlotStyle -> Hue[.0]];
```

```

plot10000 = Plot[ConditionalReliability[LogNormalDistribution[muSA, sigmaSA], t, 10000],
  {t, 10000, 70000}, DisplayFunction -> Identity, PlotStyle -> Hue[.15]];

plot20000 = Plot[ConditionalReliability[LogNormalDistribution[muSA, sigmaSA], t, 20000],
  {t, 20000, 70000}, DisplayFunction -> Identity, PlotStyle -> Hue[.3]];

plot30000 = Plot[ConditionalReliability[LogNormalDistribution[muSA, sigmaSA], t, 30000],
  {t, 30000, 70000}, DisplayFunction -> Identity, PlotStyle -> Hue[.45]];

plot40000 = Plot[ConditionalReliability[LogNormalDistribution[muSA, sigmaSA], t, 40000],
  {t, 40000, 70000}, DisplayFunction -> Identity, PlotStyle -> Hue[.6]];

plot50000 = Plot[ConditionalReliability[LogNormalDistribution[muSA, sigmaSA], t, 50000],
  {t, 50000, 70000}, DisplayFunction -> Identity, PlotStyle -> Hue[.75]];

plot60000 = Plot[ConditionalReliability[LogNormalDistribution[muSA, sigmaSA], t, 60000],
  {t, 60000, 70000}, DisplayFunction -> Identity, PlotStyle -> Hue[.9]];

Show[plotnew, plot10000, plot20000, plot30000,
  plot40000, plot50000, plot60000, Axes -> False, Frame -> True,
  FrameLabel -> {"t, miles", "Conditional Survival Probability",
    "Given Current Age 0,10000,20000,30000,40000,50000,60000 Miles", None},
  DisplayFunction -> $DisplayFunction];

```

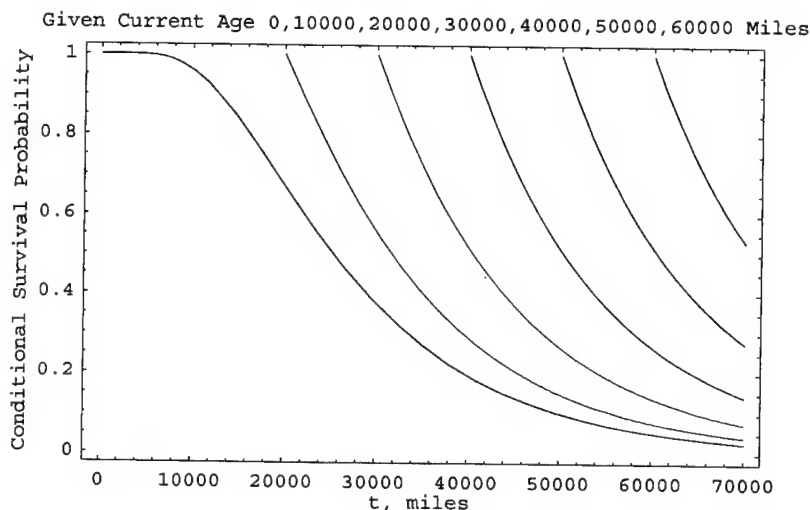


Figure 3-5

From the figure above one can see that a new shock absorber is highly reliable for approximately 10,000 miles and thereafter they gradually become unreliable. We now plot the probability of surviving the next 2,000 miles as a function of age:



```
Plot[ConditionalReliability[LogNormalDistribution[muSA, sigmaSA], age + 2000, age],
{age, 0, 70000}, Axes → False, Frame → True, FrameLabel → {"Current Mileage",
"Conditonal Survival Probability", "Survival Probability Next 2,000 miles", None},
PlotStyle → RGBColor[0, 1, 0], PlotRange → {0, 1}];
```

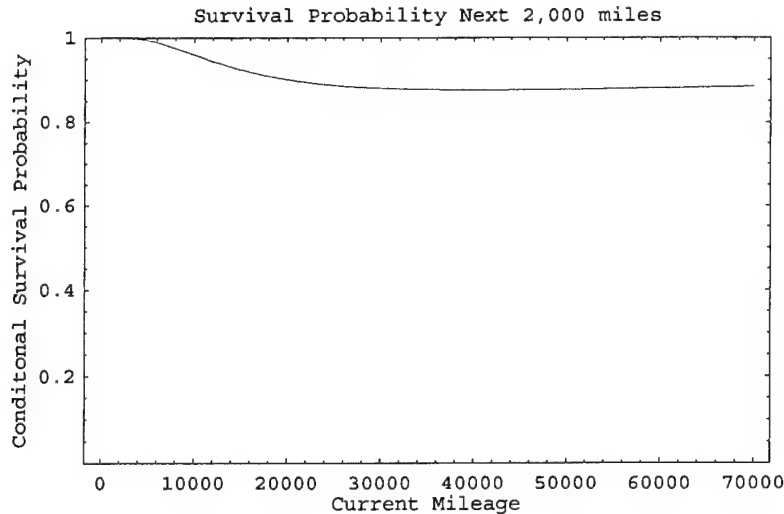


Figure 3-6

Inspection of figure 2-5 reveals the surprising result that after 20,000 miles or so, the probability of a shock absorber surviving the next 2,000 miles (given that it has survived to the start of the interval) levels off at approximately 0.9. This is due once again to the fact that the hazard function does not increase monotonically. Rather, it increases and then decreases. A table of such values is generated thus:

```
TableForm[
Table[{age, ConditionalCDF[LogNormalDistribution[muSA, sigmaSA], age + 2000, age],
ConditionalReliability[LogNormalDistribution[muSA, sigmaSA], age + 2000, age]},
{age, 0, 70000, 5000}], TableHeadings →
{None, {"Age(miles)", "CDF(next 2000)", "R(next 2000)"}, TableAlignments → Center]
```

Age(miles)	CDF(next 2000)	R(next 2000)
0	$8.34545 \times 10^{-7}$	0.999999
5000	0.00652615	0.993474
10000	0.0411984	0.958802
15000	0.0768332	0.923167
20000	0.100019	0.899981
25000	0.113327	0.886673
30000	0.120464	0.879536
35000	0.123885	0.876115
40000	0.12506	0.87494
45000	0.124861	0.875139
50000	0.123814	0.876186
55000	0.122248	0.877752
60000	0.120368	0.879632
65000	0.118308	0.881692
70000	0.116154	0.883846

The CDF column generated in a earlier section of this chapter was also included.

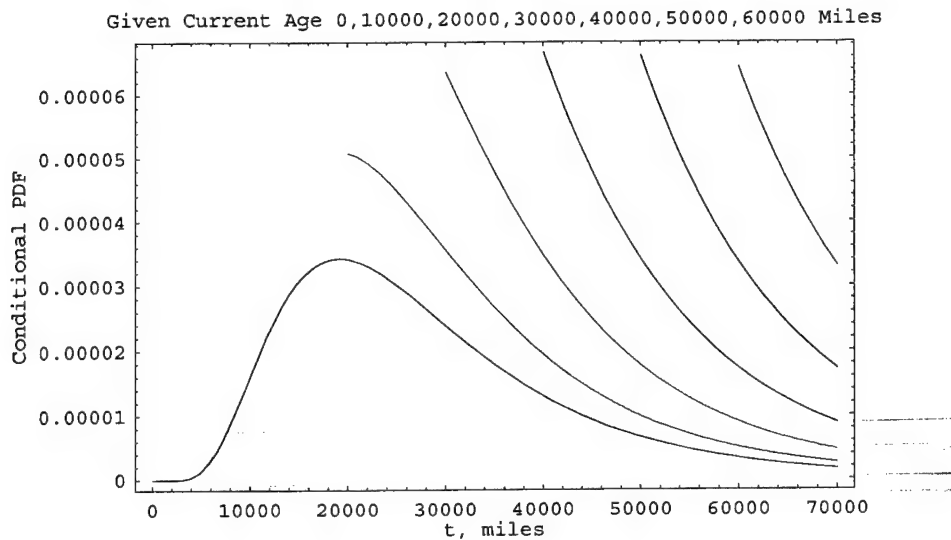
---

## Conditional PDF

Next the conditional probability density function (PDF) will be considered. A plot that generates a family of PDF curves for shock absorbers of various ages is generated thus:

```
plotnew = Plot[ConditionalPDF[LogNormalDistribution[muSA, sigmaSA], t, 0],  
  {t, 0, 70000}, DisplayFunction -> Identity, PlotStyle -> Hue[.0]];  
  
plot10000 = Plot[ConditionalPDF[LogNormalDistribution[muSA, sigmaSA], t, 10000],  
  {t, 10000, 70000}, DisplayFunction -> Identity, PlotStyle -> Hue[.15]];  
  
plot20000 = Plot[ConditionalPDF[LogNormalDistribution[muSA, sigmaSA], t, 20000],  
  {t, 20000, 70000}, DisplayFunction -> Identity, PlotStyle -> Hue[.3]];  
  
plot30000 = Plot[ConditionalPDF[LogNormalDistribution[muSA, sigmaSA], t, 30000],  
  {t, 30000, 70000}, DisplayFunction -> Identity, PlotStyle -> Hue[.45]];  
  
plot40000 = Plot[ConditionalPDF[LogNormalDistribution[muSA, sigmaSA], t, 40000],  
  {t, 40000, 70000}, DisplayFunction -> Identity, PlotStyle -> Hue[.6]];  
  
plot50000 = Plot[ConditionalPDF[LogNormalDistribution[muSA, sigmaSA], t, 50000],  
  {t, 50000, 70000}, DisplayFunction -> Identity, PlotStyle -> Hue[.75]];  
  
plot60000 = Plot[ConditionalPDF[LogNormalDistribution[muSA, sigmaSA], t, 60000],  
  {t, 60000, 70000}, DisplayFunction -> Identity, PlotStyle -> Hue[.9]];
```

```
Show[plotnew, plot10000, plot20000, plot30000, plot40000, plot50000, plot60000,
  Axes → False, Frame → True, FrameLabel → {"t, miles", "Conditional PDF",
    "Given Current Age 0,10000,20000,30000,40000,50000,60000 Miles", None},
  DisplayFunction → $DisplayFunction];
```



**Figure 3-7**

The curve that starts at zero on the mileage axis corresponds to a new shock absorber. The further to the right the curve starts, the older the component is. Figure 3-6 shows that shock absorbers that are older have increased probability density. The PDF is essentially re-scaled by component age so that the area beneath it equals one. It is difficult, however, to obtain any specific quantitative insight concerning when to replace shock absorbers of various ages.

---

## Conditional Mean Life

The next function to be considered is the conditional mean life. We can generate a graph of the conditional mean life of the track centerguide as a function of its age thus:

```
Plot[ConditionalMeanLife[LogNormalDistribution[muSA, sigmaSA], age],
{age, 0, 70000}, PlotRange -> All, Axes -> False, Frame -> True, FrameLabel ->
{"age,miles", "Mean Life", StringForm["Lognormal Log Mean = `", Std Dev = `",
muSA, sigmaSA], None}, PlotStyle -> RGBColor[.5, 0, .5]];
```

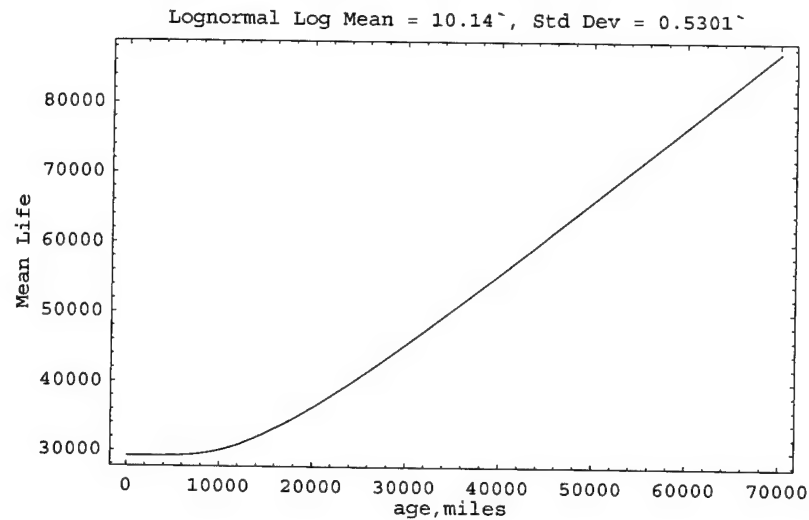


Figure 3-8

One insight from the graph above is that once a shock absorber has survived for approximately 20,000 miles, it can thereafter be expected to last another 15,000 miles regardless of age. This is an unfortunate behavior of the lognormal distribution and should not be relied upon after the hazard function begins declining at 30,000 miles.

Next, let us tabulate values of shock absorber mean life.

```
TableForm[Table[{age, ConditionalMeanLife[LogNormalDistribution[muSA, sigmaSA], age]],
{age, 0, 70000, 5000}},
TableHeadings -> {None, {"Age(miles)", "Mean Life (miles)"}}, TableAlignments -> Center]
```

Age(miles)	Mean Life (miles)
0	29158.5
5000	29185.9
10000	30025.4
15000	32530.4
20000	36240.3
25000	40626.3
30000	45390.3
35000	50375.7
40000	55497.7
45000	60707.7
50000	65976.5
55000	71285.5
60000	76622.7
65000	81980.
70000	87351.8

The table confirms the insights obtained from the graph. While the mean life function was interesting, the mean life remaining function may offer additional insight.

## Conditional Mean Life Remaining

The final new function to be considered in this chapter is the conditional mean life remaining function. We can plot the mean life remaining for the shock absorber thus:

```
Plot[ConditionalMeanLifeRemaining[LogNormalDistribution[muSA, sigmaSA], age],
{age, 0, 70000}, Axes → False, Frame → True,
FrameLabel → {"age,miles", "Mean Life Remaining",
StringForm["Lognormal Log Mean = `", Std Dev = `", muSA, sigmaSA], None},
PlotStyle → RGBColor[.5, 0, .5], PlotRange → {0, 30000}];
```

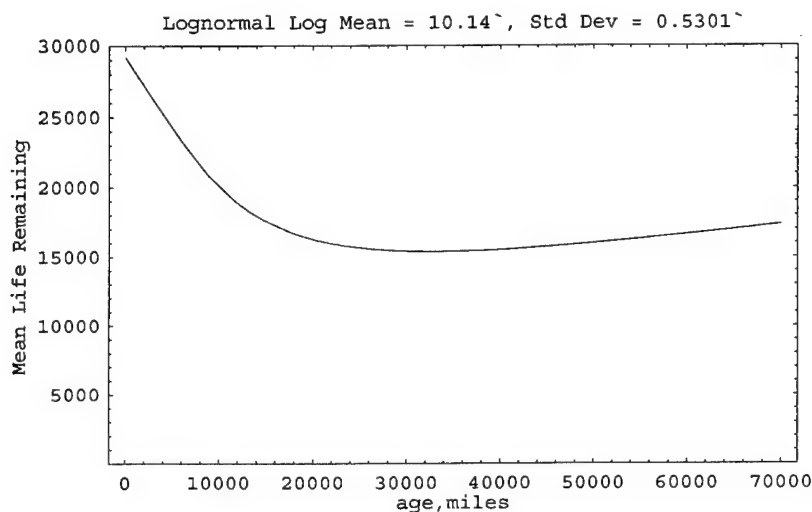


Figure 3-9

Examination of the graph above reveals that the expected life remaining decreases until approximately 30,000 miles, thereafter it actually increases steadily. This is once again due to unfortunate behavior of the hazard function which begins to decline at this point. The mean life remaining curve should not be relied upon after 30,000 miles or so.

Tabulated mean life remaining values are generated as follows:

```
TableForm[
  Table[{age, ConditionalMeanLifeRemaining[LogNormalDistribution[muSA, sigmaSA], age]},
    {age, 0, 70000, 5000}], TableHeadings ->
    {None, {"Age(miles)", "Mean Life Remaining(miles)"}, TableAlignments -> Center]
```

Age(miles)	Mean Life Remaining(miles)
0	29158.5
5000	24185.9
10000	20025.4
15000	17530.4
20000	16240.3
25000	15626.3
30000	15390.3
35000	15375.7
40000	15497.7
45000	15707.7
50000	15976.5
55000	16285.5
60000	16622.7
65000	16980.
70000	17351.8

Perhaps it would be helpful to generate a table with columns for mean life remaining and percentage of mean life remaining compared to the mean life of a new centerguide:

```
TableForm[
  Table[{age, ConditionalMeanLifeRemaining[LogNormalDistribution[muSA, sigmaSA], age],
    100 * (ConditionalMeanLifeRemaining[LogNormalDistribution[muSA, sigmaSA], age]) /
    ConditionalMeanLife[LogNormalDistribution[muSA, sigmaSA], 0]},
    {age, 0, 70000, 5000}], TableHeadings -> {None, {"Age(miles)", "Mean Life Remaining",
    "% Mean Life Remaining (New)"}, TableAlignments -> Center]
```

Age(miles)	Mean Life Remaining	% Mean Life Remaining (New)
0	29158.5	100.
5000	24185.9	82.9462
10000	20025.4	68.6775
15000	17530.4	60.1211
20000	16240.3	55.6965
25000	15626.3	53.5909
30000	15390.3	52.7814
35000	15375.7	52.7314
40000	15497.7	53.1499
45000	15707.7	53.8701
50000	15976.5	54.7917
55000	16285.5	55.8514
60000	16622.7	57.008
65000	16980.	58.2334
70000	17351.8	59.5084

It may be more appropriate to replace the percentage of mean life remaining when compared to the mean life of a component of the same age:

```
TableForm[
Table[{age, ConditionalMeanLifeRemaining[LogNormalDistribution[muSA, sigmaSA], age],
100 * (ConditionalMeanLifeRemaining[LogNormalDistribution[muSA, sigmaSA], age]) /
ConditionalMeanLife[LogNormalDistribution[muSA, sigmaSA], age]},
{age, 0, 70000, 5000}], TableHeadings -> {None, {"Age(miles)", "Mean Life Remaining",
"% Mean Life Remaining (Used)"}}, TableAlignments -> Center]
```

Age(miles)	Mean Life Remaining	% Mean Life Remaining (Used)
0	29158.5	100.
5000	24185.9	82.8684
10000	20025.4	66.6948
15000	17530.4	53.8893
20000	16240.3	44.8128
25000	15626.3	38.4635
30000	15390.3	33.9065
35000	15375.7	30.5221
40000	15497.7	27.925
45000	15707.7	25.8744
50000	15976.5	24.2154
55000	16285.5	22.8454
60000	16622.7	21.6942
65000	16980.	20.7124
70000	17351.8	19.8643

The percentage of mean life remaining column was changed very little. Finally, we can include columns for the conditional CDF and reliability that appeared earlier in this chapter to the table above:

```
TableForm[
Table[{age, ConditionalCDF[LogNormalDistribution[muSA, sigmaSA], age + 2000, age],
ConditionalReliability[LogNormalDistribution[muSA, sigmaSA], age + 2000, age],
ConditionalMeanLifeRemaining[LogNormalDistribution[muSA, sigmaSA], age],
100 * (ConditionalMeanLifeRemaining[LogNormalDistribution[muSA, sigmaSA], age]) /
ConditionalMeanLife[LogNormalDistribution[muSA, sigmaSA], age]},
{age, 0, 70000, 5000}], TableHeadings -> {None, {"Age(miles)", "CDF(next 2K)",
"R(next 2K)", "Mean Life Remain.", "% Mean Life Remain. (Used)"}},
TableAlignments -> Center, TableSpacing -> {1, 1.5}]
```

Age(miles)	CDF(next 2K)	R(next 2K)	Mean Life Remain.	% Mean Life Remain. (Used)
0	$8.34545 \times 10^{-7}$	0.999999	29158.5	100.
5000	0.00652615	0.993474	24185.9	82.8684
10000	0.0411984	0.958802	20025.4	66.6948
15000	0.0768332	0.923167	17530.4	53.8893
20000	0.100019	0.899981	16240.3	44.8128
25000	0.113327	0.886673	15626.3	38.4635
30000	0.120464	0.879536	15390.3	33.9065
35000	0.123885	0.876115	15375.7	30.5221
40000	0.12506	0.87494	15497.7	27.925
45000	0.124861	0.875139	15707.7	25.8744
50000	0.123814	0.876186	15976.5	24.2154
55000	0.122248	0.877752	16285.5	22.8454
60000	0.120368	0.879632	16622.7	21.6942
65000	0.118308	0.881692	16980.	20.7124
70000	0.116154	0.883846	17351.8	19.8643

Examination of the table above reveals that the metrics change little after around 30,000 miles. The metrics should not be relied upon after that point because the hazard function begins to decline which is not consistent with the likely failure phenomena for shock absorbers. Clearly shock absorbers do not improve with age after 30,000 miles.

Let us consider the metrics from 0 to 30,000 miles. For failure-avoidance intervals of 2,000 miles, it is highly likely that a shock absorber that has survived to the start of the interval will survive until the end of the interval. It does not appear that this shock absorber would be a promising candidate for replacement before failure-avoidance periods of 2,000 miles or less.

---

## Summary

In this chapter we set out to illustrate the new conditional distributions for the two-parameter lognormal distribution. We discovered that it is difficult to obtain much insight from a PDF graph with respect to the replacement of shock absorbers of various ages before failure. All of the other new functions appear to be valuable for evaluating the replacement of aging components before failure. These functions generate metrics, such as those in the last table above, that provide a reasonable foundation for the development of an interactive decision-making process for component replacement before failure.

Even though the shock absorber modeled in this chapter with a lognormal distribution is clearly subject to aging, it does not appear to be a good candidate for replacement before for failure-avoidance intervals of 2,000 miles or less.



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# Chapter 4

## Combined Example of Conditional Weibull and Lognormal Distributions with Multiple Components

---

### Introduction

Chapters 3 and 4 illustrated the new functions for the Weibull and lognormal distributions, respectively, for a single component of any age. In this chapter, a small but mixed group of components will be considered. This will help us visualize the information that could be available to an operator or maintainer of a piece of equipment that had such models embedded in on-board software.

---

### Parameter Values for Components

The Weibull shape and scale parameters, respectively, for the track centerguide illustrated in Chapter 2 are:

```
shapeCtrGuide = 5.14;  
scaleCtrGuide = 4602;
```

The lognormal log (base  $e$ ) mean and standard deviation parameters, respectively, for the shock absorber illustrated in Chapter 3 are:

```
muSA = 10.14;  
sigmaSA = .5301;
```

Weibull shape and scale parameters for an IDT 96 ball grid array microelectronic device that AMSAA analyzed test data for are:

```
shapeBGA = 9.52;  
scaleBGA = 5678;
```

In all cases, estimates will be treated as the true values of the parameters.

Before the new functions can be used, the add-on package `Reliability`ConditionalDistributions`` must be loaded:

```
Needs["Reliability`ConditionalDistributions`"]
```

Let us also load the standard add-on package Graphics`Legend` which will allow us to add legends to plots:

```
Needs["Graphics`Legend`"]
```

---

## Pre-Deployment Failure Probabilities

First let us assume that the components are all new and plot their cumulative distribution functions (CDF):

```
Plot[{ConditionalCDF[WeibullDistribution[shapeCtrGuide, scaleCtrGuide], t, 0],  
      ConditionalCDF[LogNormalDistribution[muSA, sigmaSA], t, 0],  
      ConditionalCDF[WeibullDistribution[shapeBGA, scaleBGA], t, 0]},  
     {t, 0, 15000}, Axes → False, Frame → True, FrameLabel → {"t, miles",  
      "Conditional Failure Probability", "Assuming All Components New", None},  
     PlotStyle → {Hue[.0], Hue[.2], Hue[.4]}, PlotRange → All,  
     PlotLegend → {"CtrGuide", "SA", "BGA"}, LegendPosition → {.95, -.4}];
```

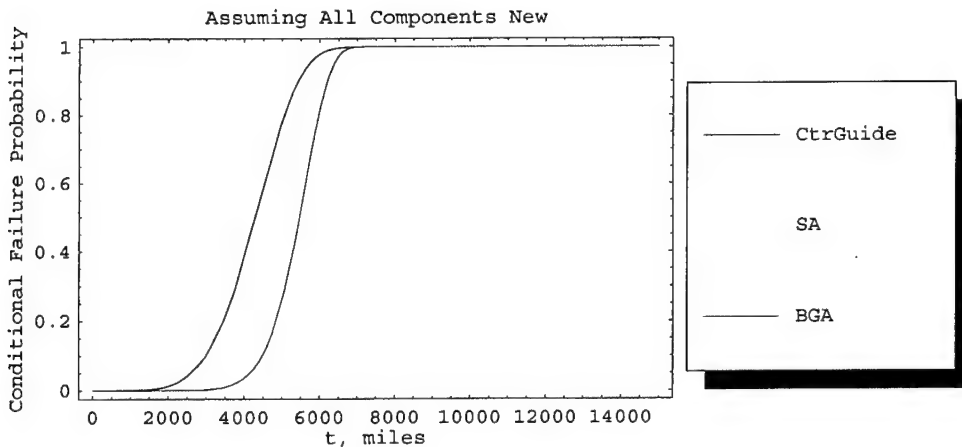


Figure 4-1

The graph above shows that if all of the components were new, the centerguide would be the first component to fail, followed closely by the BGA. The shock absorber would be of concern much later.

Generally, components will not all be new. Let us assume that at present, the components are installed on a vehicle whose odometer reads 50,000 miles.

```
odometerCurrent = 50000;
```

Let us further assume that there are twelve centerguides, four were installed at 46,000 miles, four more were installed at 47,000 miles and the final four were installed at 48,000 miles.

```
installCG1st4 = 46000;
```

```
installCG2nd4 = 47000;
```

```
installCG3rd4 = 48000;
```

Let us assume that there are four shock absorbers and they are all original to the vehicle.

```
installSA4 = 0;
```

Finally, let us assume that there is one BGA and it was installed at 46,000 miles.

```
installBGA = 46000;
```

Let us assume that the vehicle will soon be deployed and it is important to avoid failures for the next 2,000 miles since the logistics footprint will be heavily constrained. We can plot the failure probabilities for these components for the deployment thus:

```
Plot[{ConditionalCDF[WeibullDistribution[shapeCtrGuide, scaleCtrGuide],
  odometer - installCG1st4, odometerCurrent - installCG1st4],
  ConditionalCDF[WeibullDistribution[shapeCtrGuide, scaleCtrGuide],
  odometer - installCG2nd4, odometerCurrent - installCG2nd4],
  ConditionalCDF[WeibullDistribution[shapeCtrGuide, scaleCtrGuide],
  odometer - installCG3rd4, odometerCurrent - installCG3rd4],
  ConditionalCDF[LogNormalDistribution[muSA, sigmaSA],
  odometer - installSA4, odometerCurrent - installSA4],
  ConditionalCDF[WeibullDistribution[shapeBGA, scaleBGA],
  odometer - installBGA, odometerCurrent - installBGA]},
{odometer, odometerCurrent, odometerCurrent + 2000}, Axes → False, Frame → True,
FrameLabel → {"t, miles", "Conditional Failure Probability",
  "Given Current Ages for All Components", None},
PlotStyle → {Hue[.0], Hue[.2], Hue[.4], Hue[.6], Hue[.8]}, PlotRange → All,
PlotLegend → {"CG1st4", "CG2nd4", "CG3rd4", "SA", "BGA"}, LegendPosition → {.9, -.4}];
```

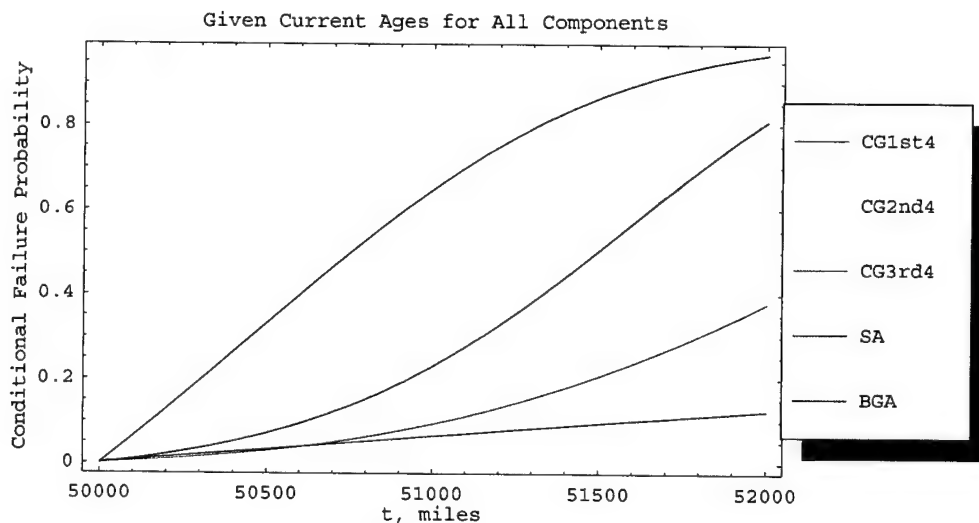


Figure 4-2

Examination of the plot above reveals that the first four centerguides are the greatest concern are virtually certain to fail during the deployment and the second four centerguides and the BGA are highly likely to fail. The last group of centerguides has approximately 40% chance of failing. We can generate a ranked table of these deployment failure probabilities as follows:

```
TableForm[Sort[tbl = Transpose[{"CtrGuide1st4 (Each)", "CtrGuide2nd4 (Each)",
  "CtrGuide3rd4 (Each)", "Shock Absorbers (Each)", "BGA"},
  {ConditionalCDF[WeibullDistribution[shapeCtrGuide, scaleCtrGuide],
    odometer - installCG1st4, odometerCurrent - installCG1st4],
  ConditionalCDF[WeibullDistribution[shapeCtrGuide, scaleCtrGuide],
    odometer - installCG2nd4, odometerCurrent - installCG2nd4],
  ConditionalCDF[WeibullDistribution[shapeCtrGuide, scaleCtrGuide],
    odometer - installCG3rd4, odometerCurrent - installCG3rd4],
  ConditionalCDF[LogNormalDistribution[muSA, sigmaSA],
    odometer - installSA4, odometerCurrent - installSA4],
  ConditionalCDF[WeibullDistribution[shapeBGA, scaleBGA],
    odometer - installBGA, odometerCurrent - installBGA]} /. odometer -> 52000]],
  {#1[[2]] > #2[[2]]} &], TableHeadings -> {None, {"COMPONENT", "FAIL PROB"}}]
```

COMPONENT	FAIL PROB
CtrGuide1st4 (Each)	0.967396
BGA	0.808919
CtrGuide2nd4 (Each)	0.758472
CtrGuide3rd4 (Each)	0.376658
Shock Absorbers (Each)	0.123814

It would probably be best to replace the first and second groups of centerguides as well as the BGA. Each of the centerguides in the third group has a 38% of failing so the probability that at least one of these will fail is (assuming shock absorbers fail independently):

```
1 - (1 - ConditionalCDF[WeibullDistribution[shapeCtrGuide, scaleCtrGuide],
  odometer - installCG3rd4, odometerCurrent - installCG3rd4] /. odometer -> 52000)^4
0.849024
```

It would probably be advisable to replace this group of centerguides as well.

Each of the shock absorbers has a 12% chance of failure. Since there are four shock absorbers, the probability that at least one will fail during the deployment (again assuming they fail independently) is:

```
1 - (1 - ConditionalCDF[LogNormalDistribution[muSA, sigmaSA],
  odometer - installSA4, odometerCurrent - installSA4] /. odometer -> 52000)^4
0.410634
```

It may or may not be advisable to replace the shock absorbers as well.

---

## Inclusion of Conditional Mean Life Remaining

It will be helpful to add columns to the table above for conditional mean life remaining and percent of mean life remaining. The values for the conditional mean life remaining column are generated thus.

```
mlr = {ConditionalMeanLifeRemaining[
  WeibullDistribution[shapeCtrGuide, scaleCtrGuide], odometerCurrent - installCG1st4],
  ConditionalMeanLifeRemaining[WeibullDistribution[shapeCtrGuide, scaleCtrGuide],
  odometerCurrent - installCG2nd4], ConditionalMeanLifeRemaining[
  WeibullDistribution[shapeCtrGuide, scaleCtrGuide], odometerCurrent - installCG3rd4],
  ConditionalMeanLifeRemaining[LogNormalDistribution[muSA, sigmaSA],
  odometerCurrent - installSA4], ConditionalMeanLifeRemaining[
  WeibullDistribution[shapeBGA, scaleBGA], odometerCurrent - installBGA]}

{831.772, 1435.38, 2267.7, 15976.5, 1454.66}
```

The values for the percent of mean life remaining column are generated thus.

```
mlrpercent =
100 * {ConditionalMeanLifeRemaining[WeibullDistribution[shapeCtrGuide, scaleCtrGuide],
  odometerCurrent - installCG1st4] / ConditionalMeanLife[WeibullDistribution[
  shapeCtrGuide, scaleCtrGuide], odometerCurrent - installCG1st4],
  ConditionalMeanLifeRemaining[WeibullDistribution[shapeCtrGuide, scaleCtrGuide],
  odometerCurrent - installCG2nd4] / ConditionalMeanLife[WeibullDistribution[
  shapeCtrGuide, scaleCtrGuide], odometerCurrent - installCG2nd4],
  ConditionalMeanLifeRemaining[WeibullDistribution[shapeCtrGuide, scaleCtrGuide],
  odometerCurrent - installCG3rd4] / ConditionalMeanLife[WeibullDistribution[
  shapeCtrGuide, scaleCtrGuide], odometerCurrent - installCG3rd4],
  ConditionalMeanLifeRemaining[LogNormalDistribution[muSA, sigmaSA],
  odometerCurrent - installSA4] / ConditionalMeanLife[
  LogNormalDistribution[muSA, sigmaSA], odometerCurrent - installSA4],
  ConditionalMeanLifeRemaining[WeibullDistribution[shapeBGA, scaleBGA],
  odometerCurrent - installBGA] / ConditionalMeanLife[
  WeibullDistribution[shapeBGA, scaleBGA], odometerCurrent - installBGA]}

{17.2146, 32.3621, 53.1363, 24.2154, 26.6682}
```

The new columns are added to the table thus:

```
TableForm[Sort[Transpose[Join[Transpose[tbl], {mlr}, {mlrpercent}]],
  (#1[[2]] > #2[[2]]) &], TableHeadings -> {None,
  {"COMPONENT", "FAIL PROB", "MEAN LIFE REMAINING", "% MEAN LIFE REMAINING (USED)"},
  TableAlignments -> Center, TableSpacing -> {1, 1.5}]
```

COMPONENT	FAIL PROB	MEAN LIFE REMAINING	% MEAN LIFE REMAINING (USED)
CtrGuide1st4 (Each)	0.967396	831.772	17.2146
BGA	0.808919	1454.66	26.6682
CtrGuide2nd4 (Each)	0.758472	1435.38	32.3621
CtrGuide3rd4 (Each)	0.376658	2267.7	53.1363
Shock Absorbers (Each)	0.123814	15976.5	24.2154

The components are still ranked by failure probability. This table shows that the top three components have expected lives shorter than the failure-avoidance period. The shock absorbers have approximately 16,000 miles of expected remaining life so this would weigh against replacing them at this time.

## Implementation in On-Board Software

Figure 4-3 depicts how the example in this chapter might look to an operator or maintainer.

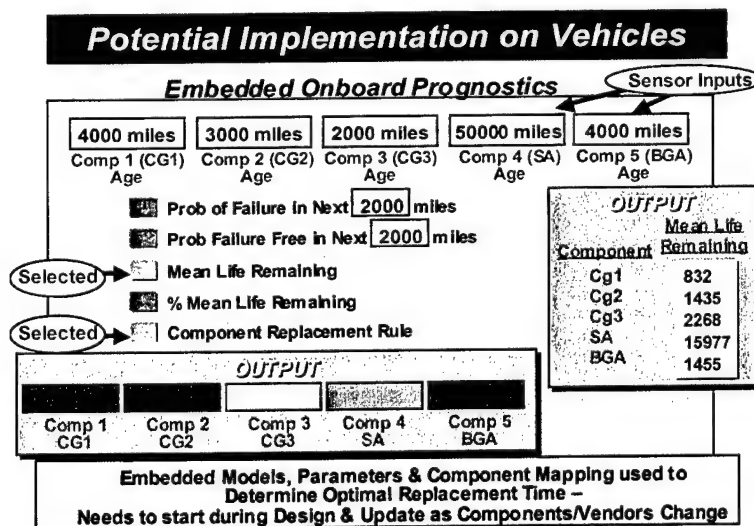


Figure 4-3

In order to embedding the prognostics approach developed in this report on-board a system, the following elements need to be addressed:

### Software:

- Models, model parameters and their mapping to components currently installed on the system.
- This needs to be started during design and updated as components or vendors change and as additional data becomes available
- Algorithms/lookup tables for distribution functions not available in closed-form (e.g., lognormal CDF and Weibull

mean life remaining).

Sensors:

- Age (miles, hours, as appropriate) of each currently-installed component covered.

User/Maintainer Input:

- Length of failure-avoidance period.
- Selection of type of results desired.

Results:

- Tables that rank components with respect to various metrics:
- Probability of failure in next \_ miles/hours.
- Mean life remaining.
- % of mean life remaining.
- Optimal replacement time.
- Green-yellow-red color coding of components.



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# Chapter 5

## Summary and Areas for Follow-on Work

This report documents the development and notional application of a new tool for developing component replacement-before-failure rules for systems of any age that are preparing for a period during which failures must be zealously avoided. The tool, which is an extension of *Mathematica*, generates graphs and tables for a variety of metrics that one should use in decision-making process. Chapters 2 and 3 illustrated use of the new functions on components whose reliability is modeled with the Weibull and lognormal distributions, respectively. Chapter 4 illustrated use of the new tool when analyzing and ranking replacement-before-failure rules for a collection of components. These chapters constitute a basic set of electronic templates for applying the new tool. A palette of buttons for the new functions was also developed to supplement the *Mathematica* graphical user interface.

Two areas were identified where further work is needed:

1. In this report, a tool was developed that generates several metrics useful for making decisions with respect to replacing components before failure. Additional work is needed in order to develop an interactive decision-making process fed by these metrics. The process should be appropriate for use by the designers, logisticians and evaluators formulating and evaluating the replace-before-failure rules. Ultimately, the tool should be extended to include the interactive decision-making process. The process should also be appropriate for embedding in on-board software that will be used by the operators and maintainers. Figure 4-3 depicts what the on-board display might look like. Elements that need to be addressed in order to implement this approach in on-board software are listed at the end of Chapter 4.

2. One potential follow-on task is to implement the tool in a series of web*Mathematica* pages. This would make the tool more user-friendly and more readily available to users because it would be available over the internet. Anyone with access to the web could use the tool. The user would not have to understand the math behind the functions they desire to use. They would be able to enter in specific parameters for their problem, and then perform a complicated calculation and generate graphs and tables by clicking a button. web*Mathematica* could facilitate the use of this tool by both developers and evaluators. Figure 5-1 depicts an example web*Mathematica* page. It could be used to generate the graph that appeared in Figure 2-3. The user would select the desired function from the list at the lower left. In this case, the user would then select the distribution and provide parameters, interval length and plotting range. The desired graph is then generated by the push of a button. The graph (or other result) can be saved to the user's PC, printed or emailed.

web*Mathematica* code needs to be developed in order to connect web pages such as that depicted above to the tool.

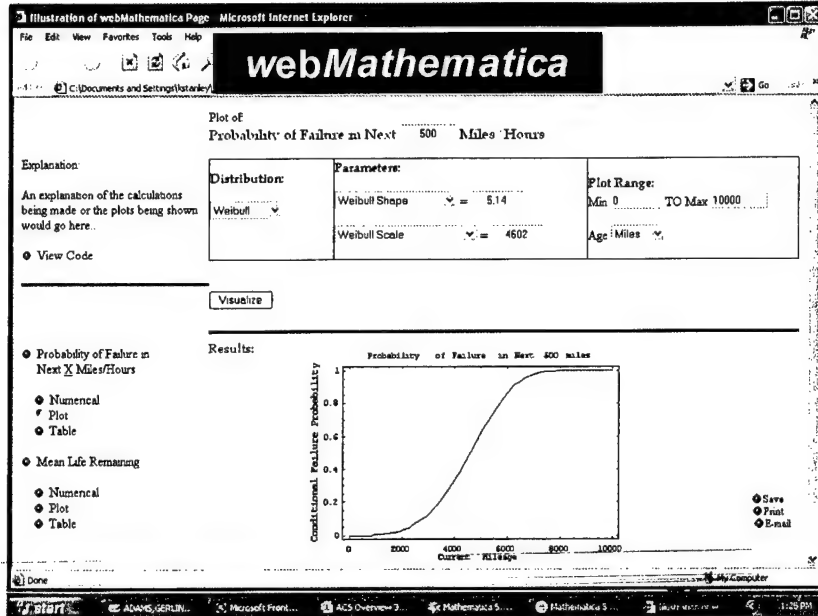


Figure 5 - 1

## *References*

Meeker W.Q., Escobar, L.A., *Statistical Methods for Reliability Data*, New York, John Wiley and Sons, 1998.

Nelson, W., *Applied Life Data Analysis*, New York, John Wiley and Sons, 1982.

O'Connor, P.D.T, *Practical Reliability Engineering*, 2nd edition, New York, John Wiley and Sons.

Wolfram, S., *The Mathematica Book*, 5th edition, Wolfram Media, 2003.

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# Appendix A

## *Conditional Distribution Functions*

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# Appendix A

## *Conditional Distribution Functions*

This notebook contains functions for the conditional Weibull and lognormal distributions thereby extending the standard add-on package *Statistics`ContinuousDistributions`*.

### Reference

#### *Title*

*Conditional Distribution Functions*

#### *Author*

Michael J. Cushing, Ph.D. and Kristin R. Stanley

#### *Summary*

This notebook contains functions for the conditional Weibull and lognormal distributions thereby extending the standard add-on package *Statistics`ContinuousDistributions`*.

#### *Copyright*

Not copyrighted.

#### *Notebook Version*

1.0

#### *Mathematica Version*

5.0



### *History*

Version 0.1.0, Jul 2003, was the initial version and it included most but not all of the Weibull functions. It was used to generate plots and tables for the 29 Jul 2003 briefing to AEC.

Version 0.5.0, 22 Sep 2003, included all of the Weibull and lognormal functions. It was used to generate the annotated briefings given to the ACS PMO on 30 Sep 2003 and the Director of the Army Evaluation Center on 10 Oct 2003.

Version 1.0, 14 Oct 2003, is functionally identical to version 0.5. Only text cells were modified.

### *Keywords*

reliability, conditional reliability, conditional Weibull distribution, conditional lognormal distribution

### *Source*

Nelson, W., *Applied Life Data Analysis*, pp. 56-71, John Wiley & Sons, 1982.

### *Warnings*

Note: all cells marked as "InitializationCell" will be written to the Auto-Save package. This package can then be read in programs that use it with `Needs["Reliability`ConditionalDistributions`"]`. Cells not intended to belong to the package do not have this property.

### *Limitation*

None known at this time.

### *Discussion*

Not applicable.

### *Requirements*

***Statistics`ContinuousDistributions`***

## **Interface**

This part declares the publicly visible functions, options, and values.

■ Set up the package context, including public imports

```
BeginPackage["Reliability`ConditionalDistributions`",  
"Statistics`ContinuousDistributions`",  
"Statistics`NormalDistribution`",  
"Statistics`DescriptiveStatistics`", "Statistics`Common`DistributionsComm  
on`"]
```

■ Usage messages for the exported functions and the context itself

The usage message for the package:

```
ConditionalDistributions::usage = "ConditionalDistributions.m (version  
1.0) is a package that contains conditional distributions for the  
Weibull and lognormal distributions thereby supplementing many of the  
Weibull and lognormal functions in the standard add-on package  
Statistics`ContinuousDistributions."
```

The usage messages for the new functions:

```
ConditionalCDF::usage = "ConditionalCDF[distribution, t, tprime] gives  
the probability using the specified distribution that an item which has  
reached the age tprime will fail by time t."
```

```
ConditionalReliability::usage = "ConditionalReliability[distribution,  
t, tprime] gives the probability using the specified distribution that  
an item which has reached the age tprime will survive to time t."
```

```
ConditionalQuantile::usage = "ConditionalQuantile[distribution, tprime,  
q] gives the qth quantile using the specified distribution for an item  
that has survived to age tprime."
```

```
ConditionalPDF::usage = "ConditionalPDF[distribution, t, tprime] gives  
the probability density function evaluated at t for an item which has  
reached the age tprime using the specified distribution."
```

```
ConditionalHazard::usage = "ConditionalHazard[distribution, t] gives  
the hazard function evaluated at t for an item using the specified  
distribution. The conditional hazard is unaffected by the age of the  
item"
```

```
ConditionalMeanLife::usage = "ConditionalMeanLife[distribution, tprime]  
gives the conditional mean age at failure for an item which has reached  
the age tprime using the specified distribution."
```

```
ConditionalMeanLifeRemaining::usage =
"ConditionalMeanLifeRemaining[distribution, tprime] gives the
conditional mean life remaining at failure for an item which has
reached the age tprime using the specified distribution."
```

- **Error messages for the exported objects**

Error trapping and messages have not been incorporated yet.

## Implementation

This part contains the actual definitions and any auxiliary functions that should not be visible outside.

- **Begin the private context (implementation part)**

```
Begin["`Private`"]
```

- **Read in any hidden imports**

None.

- **Unprotect any system functions for which definitions will be made**

We must unprotect the Weibull and lognormal definitions contained in the standard add-on package *ContinuousDistributions.m* before we can supplement them.

```
protected = Unprotect[ WeibullDistribution, LogNormalDistribution ]
```

- **Definition of auxiliary functions and local (static) variables**

None.

- **Definition of the exported functions**

*Conditional Weibull CDF*

Equation 9.44 from Nelson.

```
WeibullDistribution /: ConditionalCDF[WeibullDistribution[shape_,
scale_],
t_, tprime_] /; t >= tprime :=
1 - E^(-(tprime/scale)^shape - (t/scale)^shape)
```

#### *Conditional Weibull Reliability*

Equation 9.45 from Nelson.

```
WeibullDistribution /:  
  ConditionalReliability[WeibullDistribution[shape_, scale_], t_,  
    tprime_] /;  
  t >= tprime := E^((tprime/scale)^shape - (t/scale)^shape)
```

#### *Conditional Weibull Quantile*

Equation 9.46 from Nelson.

```
WeibullDistribution /: ConditionalQuantile[WeibullDistribution[shape_,  
  scale_], tprime_, q_] :=  
  scale*Log[1/(1 - (1 - (1 - q)*Exp[-(tprime/scale)^shape]))]^(1/shape)
```

#### *Conditional Weibull PDF*

Equation 9.47 from Nelson.

```
WeibullDistribution /: ConditionalPDF[WeibullDistribution[shape_,  
  scale_],  
    t_, tprime_] /; t >= tprime :=  
  ((shape*t^(shape - 1))*E^((tprime/scale)^shape - (t/scale)^shape))/  
  scale^shape
```

#### *Conditional Weibull Mean Life*

Equation 9.48 from Nelson.

```
WeibullDistribution /: ConditionalMeanLife[WeibullDistribution[shape_,  
  scale_], tprime_] := scale*E^(tprime/scale)^shape*  
  (Gamma[1 + 1/shape] - Gamma[1 + 1/shape, 0, (tprime/scale)^shape])
```

#### *Conditional Weibull Mean Life Remaining*

Equation 9.48 from Nelson.

```
WeibullDistribution /: ConditionalMeanLifeRemaining[  
  WeibullDistribution[shape_, scale_], tprime_] :=  
  scale*E^(tprime/scale)^shape*(Gamma[1 + 1/shape] -  
    Gamma[1 + 1/shape, 0, (tprime/scale)^shape]) - tprime
```

#### Conditional Weibull Hazard

Equation 4.1 from Nelson, p. 39. This is the unconditional Weibull hazard function. The conditional hazard function is always the same as the unconditional.

```
WeibullDistribution /: ConditionalHazard[WeibullDistribution[shape_,
scale_],
  t_] := (shape/scale)*(t/scale)^(shape - 1)
```

#### Conditional LogNormal CDF

Equation 9.2 from Nelson.

```
LogNormalDistribution /: ConditionalCDF[LogNormalDistribution[mu_,
sigma_],
  t_, tprime_] /; t >= tprime :=
  (CDF[LogNormalDistribution[mu, sigma], t] -
   CDF[LogNormalDistribution[mu, sigma], tprime])/
  (1 - CDF[LogNormalDistribution[mu, sigma], tprime])
```

#### Conditional LogNormal Reliability

Equation 9.3 from Nelson.

```
LogNormalDistribution /:
  ConditionalReliability[LogNormalDistribution[mu_, sigma_], t_,
tprime_] /;
  t >= tprime := 1 - (CDF[LogNormalDistribution[mu, sigma], t] -
  CDF[LogNormalDistribution[mu, sigma], tprime])/
  (1 - CDF[LogNormalDistribution[mu, sigma], tprime])
```

#### Conditional LogNormal Quantile

Equation 9.4 from Nelson.

```
q = (CDF[LogNormalDistribution[mu, sigma], tq] -
  CDF[LogNormalDistribution[mu, sigma], tprime]) /
  (1 - CDF[LogNormalDistribution[mu, sigma], tprime])

q == 
$$\frac{\frac{1}{2} \left( -1 - \operatorname{Erf} \left[ \frac{-\mu + \log[tprime]}{\sqrt{2} \sigma} \right] \right) + \frac{1}{2} \left( 1 + \operatorname{Erf} \left[ \frac{-\mu + \log[tq]}{\sqrt{2} \sigma} \right] \right)}{1 + \frac{1}{2} \left( -1 - \operatorname{Erf} \left[ \frac{-\mu + \log[tprime]}{\sqrt{2} \sigma} \right] \right)}$$

```

Using Solve to get a solution for tq:

```
Solve[%, tq]
```

```
- Solve::ifun :
```

*Inverse functions are being used by Solve, so some solutions may not be found.*

```
{ { tq -> emu+sqrt(2) sigma InverseErf[0,q+Erf[-mu+Log[tprime]/sqrt(2) sigma]-q Erf[-mu+Log[tprime]/sqrt(2) sigma]] }
```

Extracting the solution:

```
tq /. First[First[%]]
```

```
emu+sqrt(2) sigma InverseErf[0,q+Erf[-mu+Log[tprime]/sqrt(2) sigma]-q Erf[-mu+Log[tprime]/sqrt(2) sigma]]
```

Defining the function with this solution:

```
LogNormalDistribution /. ConditionalQuantile[LogNormalDistribution[mu_,
sigma_], tprime_, q_] :=
E^(mu + Sqrt[2]*sigma*InverseErf[0,
q + Erf[(-mu + Log[tprime])/(Sqrt[2]*sigma)] -
q*Erf[(-mu + Log[tprime])/(Sqrt[2]*sigma)]])
```

*Conditional LogNormal PDF*

Equation 9.1 from Nelson.

```
LogNormalDistribution /. ConditionalPDF[LogNormalDistribution[mu_,
sigma_],
t_, tprime_] /; t >= tprime := PDF[LogNormalDistribution[mu,
sigma], t]/
(1 - CDF[LogNormalDistribution[mu, sigma], tprime])
```

*Conditional LogNormal Mean Life*

Equation 9.6 from Nelson:

```

Integrate[t PDF[LogNormalDistribution[mu, sigma], t],
  {t, tprime, ∞}, Assumptions → tprime ≥ 0] /
(1 - CDF[LogNormalDistribution[mu, sigma], tprime])


$$\left( e^{\mu + \frac{\sigma^2}{2}} \left( \sqrt{\frac{1}{\sigma^2}} \sigma + \sqrt{\frac{1}{\sigma^2}} \sigma \operatorname{Erf} \left[ \frac{\sqrt{\frac{1}{\sigma^2}} (\mu + \sigma^2)}{\sqrt{2}} \right] - \right. \right.$$


$$\left. \left. \operatorname{Erf} \left[ \frac{\mu + \sigma^2}{\sqrt{2} \sigma} \right] + \operatorname{Erf} \left[ \frac{\mu + \sigma^2 - \operatorname{Log}[tprime]}{\sqrt{2} \sigma} \right] \right) \right) /$$


$$\left( 2 \left( 1 + \frac{1}{2} \left( -1 - \operatorname{Erf} \left[ \frac{-\mu + \operatorname{Log}[tprime]}{\sqrt{2} \sigma} \right] \right) \right) \right)$$


```

Defining the function with this solution:

```

LogNormalDistribution /: ConditionalMeanLife[LogNormalDistribution[mu_,
  sigma_], tprime_] := (E^(mu + sigma^2/2)*(Sqrt[1/sigma^2]*sigma +
  Sqrt[1/sigma^2]*sigma*Erf[(Sqrt[1/sigma^2]*(mu +
  sigma^2))/Sqrt[2]] -
  Erf[(mu + sigma^2)/(Sqrt[2]*sigma)] +
  Erf[(mu + sigma^2 - Log[tprime])/(Sqrt[2]*sigma)]))/
(2*(1 + (1/2)*(-1 - Erf[(-mu + Log[tprime])/(Sqrt[2]*sigma)])))

```

#### Conditional LogNormal Mean Life Remaining

This is the same as the ConditionalMeanLifeRemaining function except that *tprime* is subtracted from it.

```

LogNormalDistribution /: ConditionalMeanLifeRemaining[
  LogNormalDistribution[mu_, sigma_], tprime_] :=
  (E^(mu + sigma^2/2)*(Sqrt[1/sigma^2]*sigma + Sqrt[1/sigma^2]*sigma*
  Erf[(Sqrt[1/sigma^2]*(mu + sigma^2))/Sqrt[2]] -
  Erf[(mu + sigma^2)/(Sqrt[2]*sigma)] + Erf[(mu + sigma^2 -
  Log[tprime])/
  (Sqrt[2]*sigma)]))/
  (2*(1 + (1/2)*(-1 - Erf[(-mu + Log[tprime])/(Sqrt[2]*sigma)]))) -
  tprime

```

#### Conditional LogNormal Hazard

Equation 1.23 from Nelson. This is the unconditional hazard function. The conditional hazard function is always the same as the unconditional.

```

LogNormalDistribution /: ConditionalHazard[LogNormalDistribution[mu_,
  sigma_], t_] := PDF[LogNormalDistribution[mu, sigma], t]/
(1 - CDF[LogNormalDistribution[mu, sigma], t])

```

- **Definitions for system functions**

None.

- **Restore protection of system symbols**

```
Protect[ Evaluate[protected] ]
```

- **End the private context**

```
End[ ]
```

## **Epilog**

This section protects exported symbols and ends the package.

- **Protect exported symbol**

```
Protect[ ConditionalDistributions, ConditionalCDF,  
ConditionalReliability, ConditionalQuantile, ConditionalPDF,  
ConditionalHazard, ConditionalMeanLife, ConditionalMeanLifeRemaining ]
```

- **End the package context**

```
EndPackage[ ]
```



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# Appendix B

*Conditional Distributions Palette for Mathematica  
Graphical User Interface*

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# Appendix B

## *Conditional Distributions Palette for Mathematica Graphical User Interface*

First the Reliability`ConditionalDistributions` package is loaded:

```
Needs["Reliability`ConditionalDistributions`"]
```

The current version of the package is:

```
? ConditionalDistributions
```

ConditionalDistributions.m (version 1.0) is a package that contains conditional distributions for the Weibull and lognormal distributions thereby supplementing many of the Weibull and lognormal functions in the standard add-on package Statistics`ContinuousDistributions.

Here is the palette:

ConditionalCDF[WeibullDistribution[■, ■], ■, ■]
ConditionalReliability[WeibullDistribution[■, ■], ■, ■]
ConditionalQuantile[WeibullDistribution[■, ■], ■, ■]
ConditionalPDF[WeibullDistribution[■, ■], ■, ■]
ConditionalMeanLife[WeibullDistribution[■, ■], ■]
ConditionalMeanLifeRemaining[WeibullDistribution[■, ■], ■]
ConditionalHazard[WeibullDistribution[■, ■], ■]
ConditionalCDF[LogNormalDistribution[■, ■], ■, ■]
ConditionalReliability[LogNormalDistribution[■, ■], ■, ■]
ConditionalQuantile[LogNormalDistribution[■, ■], ■, ■]
ConditionalPDF[LogNormalDistribution[■, ■], ■, ■]
ConditionalMeanLife[LogNormalDistribution[■, ■], ■]
ConditionalMeanLifeRemaining[LogNormalDistribution[■, ■], ■]
ConditionalHazard[LogNormalDistribution[■, ■], ■]

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# Appendix C

## *Installation Instructions for New Tool*

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# Appendix C

## *Installation Instructions for New Tool*

Before installing the new tool, a bit of set up is required. First, one needs to create a directory named `Reliability` directly under the `ExtraPackages` directory which in turn appears within the `Add-Ons` directory. The `ConditionalDistributions` package, both the notebook (`.nb`) and executable (`.m`) files (provided as Appendix A herein), must be copied there.

A palette was developed for the tool and the files that generated the palette are provided as Appendix B. The palette is a *Mathematica* notebook named `ConditionalDistributionsPalette.nb`. The palette should be copied to the `Palettes` directory which appears in the `FrontEnd` directory within the `SystemFiles` directory. When *Mathematica* is subsequently started, the new palette will be listed in the `Palette` menu. (The `Palette` menu appears under `File` on the *Mathematica* toolbar.) Before using the palette, the `Reliability`ConditionalDistributions`` package must be loaded.

If a copy of *Mathematica* is not available, a free reader is available from the makers of *Mathematica* at [www.wolfram.com/mathreader](http://www.wolfram.com/mathreader). With this reader, one can read and print the electronic version of this report. The tool is not, however, executable without *Mathematica*.



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# Appendix D

*Checking of Conditional Distributions Package Against  
Nelson's Examples*

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# Appendix D

## *Checking of Conditional Distributions Package Against Nelson's Examples*

This appendix uses the new functions defined in Reliability`ConditionalDistributions` on the Weibull and lognormal conditional distribution examples in Nelson's text. This was done to informally verify that our functions work correctly.

### **Load the Package**

First the Reliability`ConditionalDistributions` package is loaded:

```
Needs["Reliability`ConditionalDistributions`"]
```

The current version of the package is:

```
? ConditionalDistributions
```

```
ConditionalDistributions.m (version 1.0) is a package that contains  
conditional distributions for the Weibull and lognormal distributions  
thereby supplementing many of the Weibull and lognormal functions  
in the standard add-on package Statistics`ContinuousDistributions.
```

### **Weibull Functions**

#### ■ **Parameters for Nelson's Example**

Nelson illustrates the conditional Weibull distribution with a model for generator field windings. This example can be found in pp. 69-71 of his text. The shape and scale parameters are, respectively:

```
shape = 2;
```

```
scale = 13;
```

The scale parameter is in years.

#### ■ Conditional CDF

The syntax for this function is:

**? ConditionalCDF**

ConditionalCDF[distribution, t, tprime] gives the probability using the specified distribution that an item which has reached the age tprime will fail by time t.

The probability of failure in the next two years for a winding that is 6.5 years old is:

```
ConditionalCDF[WeibullDistribution[shape, scale], 8.5, 6.5]  
0.162651
```

This agrees with Nelson's answer of 0.163.

#### ■ Conditional Quantile

The syntax for this function is:

**? ConditionalQuantile**

ConditionalQuantile[distribution, tprime, q] gives the qth quantile using the specified distribution for an item that has survived to age tprime.

The median age at failure (the 50th percentile) for a winding that is 6.5 years old is:

```
ConditionalQuantile[WeibullDistribution[shape, scale], 6.5, .5]  
12.625
```

This agrees with Nelson's answer of 12.6 years. The median for a new winding is:

```
ConditionalQuantile[WeibullDistribution[shape, scale], 0, .5]  
10.8232
```

This agrees with Nelson's answer of 10.8 years.

### ■ Conditional Reliability

The syntax for this function is:

**? ConditionalReliability**

ConditionalReliability[distribution, t, tprime] gives the probability using the specified distribution that an item which has reached the age tprime will survive to time t.

The probability of survival for the next two years for a winding that is 6.5 years old is:

```
ConditionalReliability[WeibullDistribution[shape, scale], 8.5, 6.5]
0.837349
```

This agrees with Nelson's answer of 0.837.

### ■ Conditional PDF

The syntax for this function is:

**? ConditionalPDF**

ConditionalPDF[distribution, t, tprime] gives the probability density function evaluated at t for an item which has reached the age tprime using the specified distribution.

Nelson doesn't have a numerical example for this function. We can calculate the probability of failure in the next two years for a winding that is 6.5 years old by integrating the ConditionalPDF function from 6.5 to 8.5 years. This is what was done more simply with the ConditionalCDF function above.

```
NIntegrate[
  ConditionalPDF[WeibullDistribution[shape, scale], t, 6.5], {t, 6.5, 8.5}]
0.162651
```

This agrees with Nelson's answer of 0.163.

### ■ Conditional Hazard

The syntax for this function is:

### ? ConditionalHazard

ConditionalHazard[distribution, t] gives the hazard function evaluated at t for an item using the specified distribution.

The conditional hazard is unaffected by the age of the item

Nelson doesn't have a numerical example for this function. We can calculate the probability of survival in the next two years for a winding that is 6.5 years old by integrating the ConditionalHazard function from 6.5 to 8.5 years, changing the sign of the result and then taking the base *e* antilog. This is what was done more simply with the ConditionalReliability function above.

$$e^{-\int_{6.5}^{8.5} \text{ConditionalHazard}[\text{WeibullDistribution}[\text{shape}, \text{scale}], t] dt}$$

0.837349

This agrees with Nelson's answer of 0.837.

### ■ Conditional Mean Life

The syntax for this function is:

#### ? ConditionalMeanLife

ConditionalMeanLife[distribution, tprime] gives the conditional mean age at failure for an item which has reached the age tprime using the specified distribution.

For a winding that is 6.5 years old, the mean age at failure is:

ConditionalMeanLife[WeibullDistribution[shape, scale], 6.5]

13.5933

This agrees with Nelson's answer of 13.6 years.

### ■ Conditional Mean Life Remaining

The syntax for this function is:

### ? ConditionalMeanLifeRemaining

ConditionalMeanLifeRemaining[distribution, tprime] gives the conditional mean life remaining at failure for an item which has reached the age tprime using the specified distribution.

For a winding that is 6.5 years old, the mean life remaining is:

```
ConditionalMeanLifeRemaining[WeibullDistribution[shape, scale], 6.5]
```

```
7.09334
```

This agrees with Nelson's answer of 7.1 years.

## Lognormal Functions

### ■ Parameters for Nelson's Example

Nelson illustrates the conditional lognormal distribution with a model for locomotive controls. This example can be found in pp. 67-69 of his text. Unfortunately, he uses the base 10 logarithm form of the lognormal distribution which is somewhat unusual. We use the more common base  $e$  form so we must convert his lognormal parameters first. The lognormal distribution functions defined in the standard add-on package Statistics`ContinuousDistributions` use the base  $e$  form. The package is loaded thus:

```
Needs["Statistics`ContinuousDistributions`"]
```

We need to obtain the base  $e$  log mean and log standard deviation in terms of their base 10 counterparts. We need two equations in order to solve for these two unknowns. The median for the base  $e$  form of the lognormal distribution can be calculated by the Quantile function:

```
baseemedian = Quantile[LogNormalDistribution[mue, sigmae], 1/2]
```

$e^{\text{mue}}$

The base  $e$  parameters log mean and log standard deviation are named mue and sigmae, respectively. The median for the base 10 form of the distribution is:

```
basetenmedian = 10mu10
```

$10^{\text{mu10}}$



The base ten parameters are named `mu10` and `sigma10`. Equating these medians we have the first equation we need:

```
eqn1 = baseemedian == basetenmedian;
```

The mean for the base *e* form of the lognormal distribution can be calculated by the `Mean` function:

```
baseemean = Mean[LogNormalDistribution[mue, sigmae]]
```

$$e^{\text{mue} + \frac{\text{sigmae}^2}{2}}$$

The mean for the base 10 form of the distribution can be found on page 34 of Nelson's text as equation 3.6:

$$\text{basetenmean} = 10^{\text{mu10} + \frac{\text{Log}[10] \text{sigma10}^2}{2}}$$

$$10^{\text{mu10} + \frac{1}{2} \text{sigma10}^2 \text{Log}[10]}$$

Equating these means we have the second equation we need:

```
eqn2 = baseemean == basetenmean;
```

Solving for the base *e* log mean and log standard deviation we obtain:

```
sol = Solve[{eqn1, eqn2}, {mue, sigmae}]
```

— *Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. More...*

$$\left\{ \left\{ \text{mue} \rightarrow \text{Log}[10^{\text{mu10}}], \text{sigmae} \rightarrow -\sqrt{2} \sqrt{-\text{Log}[10^{\text{mu10}}] + \text{Log}\left[10^{\text{mu10} + \frac{1}{2} \text{sigma10}^2 \text{Log}[10]}\right]} \right\}, \right. \\ \left. \left\{ \text{mue} \rightarrow \text{Log}[10^{\text{mu10}}], \text{sigmae} \rightarrow \sqrt{2} \sqrt{-\text{Log}[10^{\text{mu10}}] + \text{Log}\left[10^{\text{mu10} + \frac{1}{2} \text{sigma10}^2 \text{Log}[10]}\right]} \right\} \right\}$$

The second solution is the one that falls within the domain of the parameters. We can use the first part of it to write a function for converting a base 10 log mean to base *e*:

```
muConversion[mu10_] := Log[10mu10]
```

For Nelson's control unit example, we convert his base 10 log mean to base *e*:

```
logmu = muConversion[2.236]
```

```
5.14858
```

We can use the second part to define a function for obtaining the base  $e$  log standard deviation from the base 10 parameters:

```
sigmaConversion[mu10_, sigma10_] :=  
 $\sqrt{2} \sqrt{(-\text{Log}[10^{\text{mu10}}] + \text{Log}[10^{\text{mu10} + \frac{1}{2} \text{sigma10}^2 \text{Log}[10]}])}$ 
```

For Nelson's control unit example, we obtain the base  $e$  log standard deviation from his base 10 parameters thus:

```
logsigma = sigmaConversion[2.236, 0.320]
```

```
0.736827
```

Now we can apply our conditional lognormal functions to Nelson's example and check our answers against his.

#### ■ Conditional CDF

The syntax for this function is:

```
? ConditionalCDF
```

```
ConditionalCDF[distribution, t, tprime] gives the  
probability using the specified distribution that an  
item which has reached the age tprime will fail by time t.
```

The probability that a locomotive control unit with 240 thousand miles on it will fail in the next 20 thousand miles is:

```
ConditionalCDF[LogNormalDistribution[logmu, logsigma], 260, 240]
```

```
0.116939
```

This is quite close to Nelson's answer of 0.118. The small disagreement is likely due to the fact that he was using standard normal tables and/or to rounding at intermediate steps. The probability of failure in the next 20 thousand miles for a control unit with 120 thousand miles is:

```
ConditionalCDF[LogNormalDistribution[logmu, logsigma], 140, 120]
```

```
0.112459
```

This agrees with Nelson's answer of 0.112. The probability of failure in the next 20 thousand miles for a new control unit is:

```
ConditionalCDF[LogNormalDistribution[logmu, logsigma], 20, 0]
```

```
0.00174018
```

This agrees with Nelson's answer of 0.018.

### ■ Conditional Quantile

The syntax for this function is:

```
? ConditionalQuantile
```

```
ConditionalQuantile[distribution, tprime,  
  q] gives the qth quantile using the specified  
  distribution for an item that has survived to age tprime.
```

The median age at failure (the 50th percentile) for a control unit with 240 thousand miles on it is:

```
ConditionalQuantile[LogNormalDistribution[logmu, logsigma], 240, .5]
```

```
355.
```

This agrees with Nelson's answer of 355 thousand miles. The median for a new control unit is:

```
ConditionalQuantile[LogNormalDistribution[logmu, logsigma], 0, .5]
```

```
172.187
```

This agrees with Nelson's answer of 172 thousand miles.

### ■ Conditional Reliability

The syntax for this function is:

### **? ConditionalReliability**

ConditionalReliability[distribution, t, tprime] gives the probability using the specified distribution that an item which has reached the age tprime will survive to time t.

The probability of survival for the next 20 thousand miles for a control unit that already has 240 thousand miles on it is:

```
ConditionalReliability[LogNormalDistribution[logmu, logsigma], 260, 240]  
0.883061
```

This agrees with Nelson's answer of 0.882. The probability of survival for the next 20 thousand miles for a control unit that already has 120 thousand miles on it is:

```
ConditionalReliability[LogNormalDistribution[logmu, logsigma], 140, 120]  
0.887541
```

This agrees with Nelson's answer of 0.888. The probability of survival for the next 20 thousand miles for a new control unit is:

```
ConditionalReliability[LogNormalDistribution[logmu, logsigma], 20, 0]  
0.99826
```

This agrees with Nelson's answer of 0.9982.

### ■ **Conditional PDF**

The syntax for this function is:

#### **? ConditionalPDF**

ConditionalPDF[distribution, t, tprime] gives the probability density function evaluated at t for an item which has reached the age tprime using the specified distribution.

Nelson doesn't have a numerical example for this function. We can calculate the probability of failure in the next 20 thousand miles for a control unit that has 120 thousand miles on it already by integrating the

ConditionalPDF function from 120 to 140 thousand miles. This is what was done more simply with the ConditionalCDF function above.

```
NIntegrate[ConditionalPDF[
  LogNormalDistribution[logmu, logsigma], t, 120], {t, 120, 140}]

0.112459
```

This agrees with Nelson's answer of 0.112 obtained above.

### ■ Conditional Hazard

The syntax for this function is:

#### ? ConditionalHazard

ConditionalHazard[distribution, t] gives the hazard function evaluated at t for an item using the specified distribution. The conditional hazard is unaffected by the age of the item

Nelson doesn't have a numerical example for this function. We can calculate the probability of survival in the next 20 thousand miles for a control unit that has 240 thousand miles on it by integrating the ConditionalHazard function from 240 to 260, changing the sign of the result, and then taking the base  $e$  antilog. This is what was done more simply with the ConditionalReliability function above.

```
Exp[-NIntegrate[ConditionalHazard[
  LogNormalDistribution[logmu, logsigma], t], {t, 240, 260}]]

0.883061
```

This agrees with Nelson's answer of 0.882.

### ■ Conditional Mean Life

The syntax for this function is:

#### ? ConditionalMeanLife

ConditionalMeanLife[distribution, tprime] gives the conditional mean age at failure for an item which has reached the age tprime using the specified distribution.

For a control unit that has 240 thousand miles on it, the mean mileage at failure is:

```
ConditionalMeanLife[LogNormalDistribution[logmu, logsigma], 240]  
424.339
```

This agrees with Nelson's answer of 423. The mean mileage at failure for a new control unit is:

```
ConditionalMeanLife[LogNormalDistribution[logmu, logsigma], 0]  
225.888
```

This agrees with Nelson's answer of 225.

#### ■ Conditional Mean Life Remaining

The syntax for this function is:

**? ConditionalMeanLifeRemaining**

ConditionalMeanLifeRemaining[distribution, tprime] gives the conditional mean life remaining at failure for an item which has reached the age tprime using the specified distribution.

For a control unit that has 240 thousand miles on it, the mean life remaining is:

```
ConditionalMeanLifeRemaining[LogNormalDistribution[logmu, logsigma], 240]  
184.339
```

If we add the 240 thousand miles that the control unit already has on it we obtain a mean life of:

```
% + 240  
424.339
```

This agrees with Nelson's answer of 423.

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# Appendix E

## *Distribution List*



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# Appendix E

## *Distribution List*

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